# Geometry Teacher's Edition - Solution Key

CK-12 Foundation

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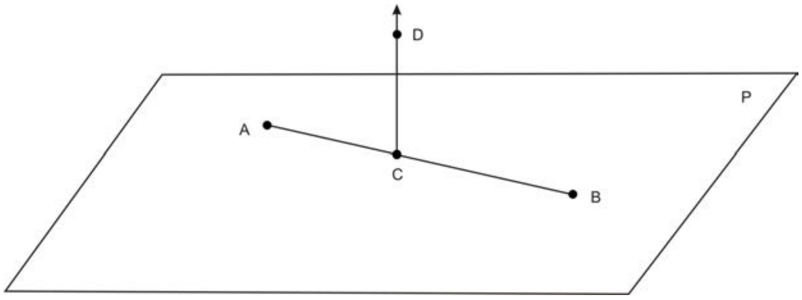
# Chapter 1

# TE Basics of Geometry - Solution Key

## 1.1 Basics of Geometry

### Points, Lines, and Planes

1. There are multiple ways to represent this diagram.

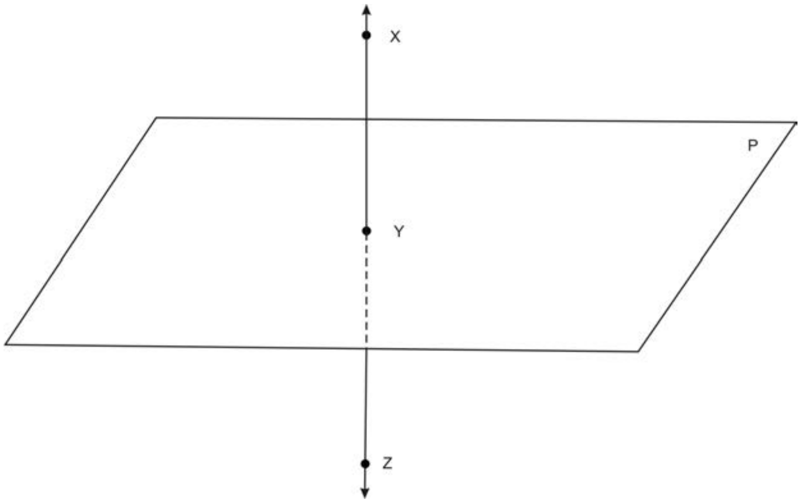


Plane $P$ contains $\overline{AB}$ but not $\overrightarrow{CD}$. $\overrightarrow{CD}$ intersects $\overline{AB}$ at $C$.

2. One can name a line by the lowercase letter associated with it, line $m$. One can use any two points on it to name the line, in any order, i.e. $\overleftrightarrow{WX}$, $\overleftrightarrow{XW}$, $\overleftrightarrow{XY}$, $\overleftrightarrow{YX}$, $\overleftrightarrow{WY}$,$\overleftrightarrow{YW}$.

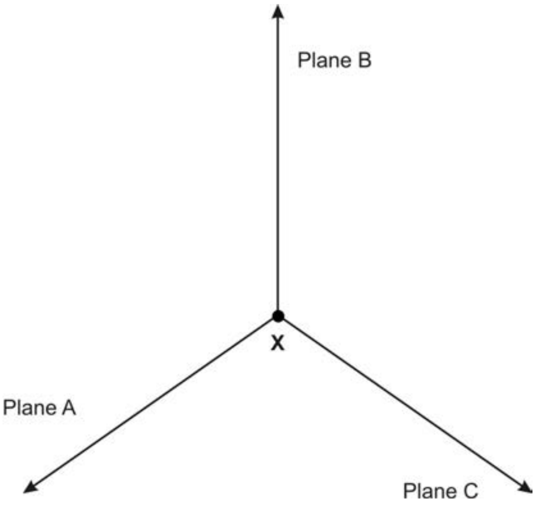
3. The best possible model is a plane because it is flat and extends in only two directions.

4. The intersection of a line and a plane is a point.



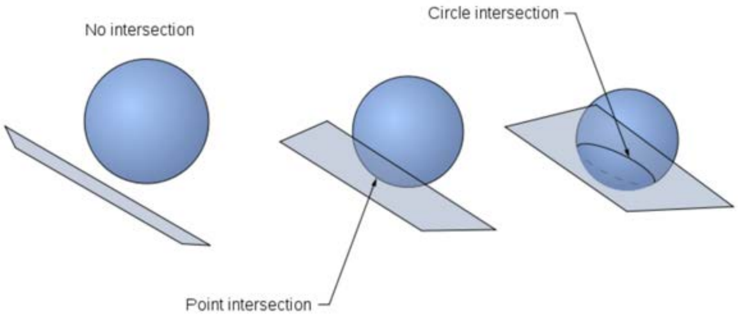
Plane $P$ and $\overleftrightarrow{XZ}$ intersect at $Y$.

5. Three planes intersect at one point. Consider two walls of your classroom and the floor.



Planes $A$, $B$, and $C$ intersect at $X$.

6. A sphere and plane intersect at a circle. Consider an orange (sphere) sliced by a knife blade (plane).



7. $\overleftrightarrow{RS}$ intersects $\overrightarrow{PQ}$ at $Q$.

8. "Any two distinct points are collinear" is a true statement. From the Line Postulate, "There is exactly one line through any two points." (refer to page 10)

9. False: The definition of a plane states that points are noncollinear to make a distinct plane. Three collinear points could be in several planes.

10. Any three noncollinear distinct points determine a plane.

## Segments and Distance

1. The distance depends on many things including resolution of printer or computer screen, margins, etc. A sample answer 3.25 inches.

2. Measuring just the body gives approximately 4.3 cm.

3. $6.5 + 4.3 = 10.8$ cm if the cockroach was facing the same direction. If it is facing the opposite direction, then $6.5 - 4.3 = 2.2$ cm.

4. Since $M$ is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$. By the definition of congruence, $PM = MQ$. By the Segment Addition Postulate (SAP), $PM+MQ = PQ$. Since $PM = MQ$, by substitution, $PM+PM = PQ$, or simplified, $2(PM) = PQ$. Therefore, since we are given that $PM = 8, 2(8) = PQ$, which tells us that $16 = PQ$.

5. By the SAP, $CD + DE = CE$, so by substitution, 3 ft + 9 ft $= CE$. Therefore, 12 ft $= CE$.

6. Using the SAP, $MN + NO = MO$. Substituting the given values gives us $x + 7 = 36$. By the subtraction property of equality, $x + 7 - 7 = 36 - 7$, which gives us $x = 29$.

7. Our two points are $(-2, 3)$ and $(-2, -7)$. If we label the first point Point 1, and the second Point 2, then we can label the coordinates as such → $x_1 = -2,\ y_1 = 3,\ x_2 = -2$ and $y_2 = -7$. If we substitute these values into the distance formula ($d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$), then we get $d = \sqrt{(-2 - (-2))^2 + (-7 - 3)^2} = \sqrt{(0)^2 + (-10)^2} = 100$. Since the line is vertical, this operation can be simplified if you notice that the $x-$coordinates cancel out. Then the distance can be calculated easily by $d = (y_2 - y_1)^2 = (-7 - (-3))^2 = 10^2 = 100$. This simpler version only works for vertical lines. For horizontal lines, the $y-$coordinates will cancel, leaving $d = (x_2 - x_1)^2$.

8. False; we must know if the points are collinear first.

9. This statement is true.

10. The points must be collinear. If $A,\ B,\ C$ are collinear, and $AB = 5$ and $BC = 12$, then $AC = 17$ by the SAP.

## Rays and Angles

1. This question is misleading because there are several rays in the diagram. I recommend adding the phrase "with initial point $C$" to the sentence so it reads "Give two possible names for the ray with initial point $C$ in the diagram." With that the possible answers are $\overrightarrow{CD}$ or $\overrightarrow{CE}$.

2. Since you can name a line by using any two points on it, you can name the line any of the following:

$$\overleftrightarrow{AB}.\ \overleftrightarrow{BA},\ \overleftrightarrow{AD},\ \overleftrightarrow{DA},\ \overleftrightarrow{BD},\ \overleftrightarrow{DB}.$$

3. $\angle DBC$, $\angle BDC$, $\angle ADE$

4. $\angle BDE$, $\angle ADC$, $\angle DCB$

5. $\angle BDA$

6. $\angle C$

7. If multiple rays, lines or segments join at the same vertex, multiple angles are formed, so it is necessary to be more specific. However, if only one angle is formed by the intersection, then it is appropriate to use just the letter of the vertex.

8. By using the Protractor Postulate, $m\angle PQR = 130^\circ - 50^\circ = 80^\circ$

9. By the Angle Addition Postulate (AAP), $m\angle FNH + m\angle HNI = m\angle FNI$, so by the Subtraction Property of Equality,

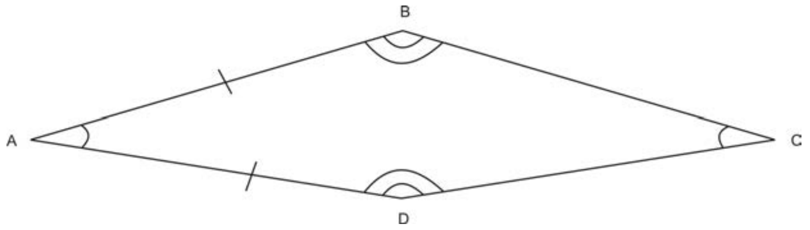
$m\angle FNH + m\angle HNI - m\angle HNI = m\angle FNI - m\angle HNI$, or $m\angle FNH = m\angle FNI - m\angle HNI$.

By substitution, $m\angle FNH = 125^\circ - 50^\circ = 75^\circ$.

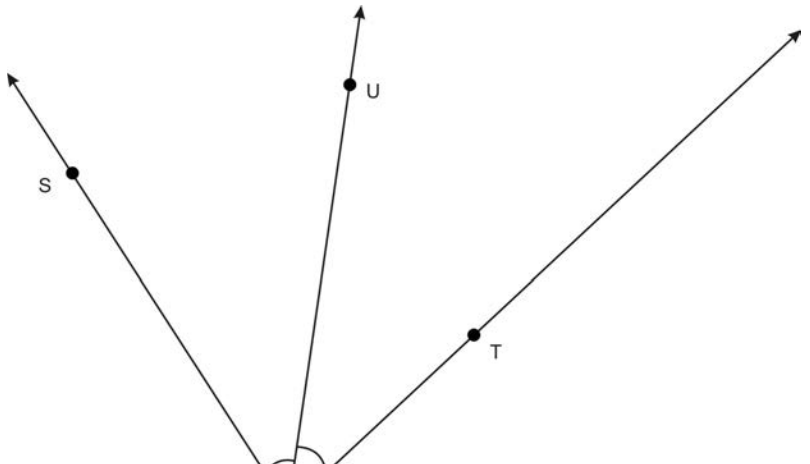
10. False; $20^\circ + 20^\circ = 40^\circ$

## Segments and Angles

1.



2.



The figure is drawn so that $m\angle SRU = m\angle TRU$, so $\angle SRU \cong \angle TRU$.

3. Since $\overrightarrow{RU}$ is the angle bisector of $\angle SRT$. By the definition of an angle bisector, $\angle SRU \cong \angle TRU$.

Therefore, by the definition of congruence, $m\angle SRU = m\angle TRU$. By the Angle Addition Postulate, we know that $m\angle SRT = m\angle SRU + \angle TRU$. Since $m\angle SRU = m\angle TRU$, we can substitute, giving us $m\angle SRT = m\angle SRU + m\angle SRU$, or $m\angle SRT = 2(m\angle SRU)$. If we know that $m\angle SRT = 64^\circ$, then $64 = 2(m\angle SRU)$. Dividing by 2 gives us $32^\circ = m\angle SRU$.

4. Since $DEFG$ is a rectangle, then opposite sides are congruent. So, $DG = 24$ in. Since $H$ is the midpoint of $DG$, it divides the segment into 2 congruent segments. By the definition of congruent segments, $GH = DH$. By the Segment Addition Postulate, $GH + DH = DG$. Using substitution gives us $24 = GH + GH$, or $24 = 2(GH)$. Diving by 2 gives us $GH = 12$ in.

5. Since $ABHG$ is a rectangle, then opposite sides are congruent, so $\overline{AB} \cong \overline{GH}$. By the definition of congruent segments, $AB = GH$. Therefore $AB = 12$ in.

6. Since $ACEF$ is a rectangle, then opposite sides are congruent, so $\overline{AC} \cong \overline{EF}$. By the definition of congruent segments, $AC = EF$. We are given in the diagram that $EF = 24$ in, so $AC = 24$ in.

7. We are given that $H$ is the midpoint of $AE$. By the definition of midpoint, $\overline{HA} \cong \overline{HE}$. By the definition of congruent segments, $HA = HE$. Since $HA = 13$, $HE = 13$ in.

8. By the Segment Addition Postulate, $HA + HE = AE$. Using substitution, $13 + 13 = AE$, so $AE = 26$ in.

9. Since $BCDH$ is a rectangle, then opposite sides are congruent, so $\overline{BH} \cong \overline{DC}$. By the definition of congruent segments, $BH = DC$. We are given in the diagram that $BH = 5$ in, so $DC = 5$ in. $D$ is the midpoint of $EC$. By the definition of midpoint, $\overline{DC} \cong \overline{ED}$. By the definition of congruent segments, $DC = ED$. Since $DC = 5$ in, $ED = 5$ in. The Segment Addition Postulate tells us that $DC + ED = EC$. By substitution, $EC = 5 + 5 = 10$ in.

10. Since $GDEF$ is a rectangle, then opposite sides are congruent, so $\overline{ED} \cong \overline{GF}$. By the definition of congruent segments, $ED = GF$. Therefore, since $ED = 5$ in, $GF = 5$ in.

11. The answer lies in the fact that rectangles have opposite sides congruent. There are 2 triangles in rectangle $ABHG$. Since $ABHG \cong BCDH$, then 2 triangles will fit inside it. Since $ACDG \cong GDEF$, the same number of triangles fit into each one, 4. So, $4 + 4 = 8$. Eight triangles fit inside $ACEF$.

## Angle Pairs

1. The definition of complementary angles is two angles that have a sum of 90°. So, The complement to $\angle A$ can be found by the equation complement $= 90 - m\angle A$. This equation can be used for $a - d$.

a. $90 - 45 = 45^\circ$

b. $90 - 82 = 8^\circ$

c. $90 - 19 = 71^\circ$

d. Since we do not know the exact value of $z$, we can only simplify to $90 - z$.

2. The definition of supplementary angles is two angles that have a sum of 180°. So, The supplement to $\angle B$ can be found by the equation complement $= 180 - m\angle B$. This equation can be used for $a - d$.

a. $180 - 45 = 135^\circ$

b. $180 - 118 = 62^\circ$

c. $180 - 32 = 148^\circ$

d. Since we do not know the exact value of $x$, we can only simplify to $180 - x$.

3. $\angle ABD$ and $\angle DBC$ are supplementary. By the definition of supplementary, $m\angle ABD + m\angle DBC = 180^\circ$. By substitution, $(3x + 7) + (5x - 3) = 180$.

This gives $8x + 4 = 180$

$$
\begin{aligned}
8x &= 176 \\
x &= 22.
\end{aligned}
$$

By substitution, $m\angle ABD = 3(22) + 7 = 73^\circ$.

By substitution, $m\angle DBC = 5(22) - 3 = 107^\circ$.

As a check, $73 + 107 = 180^\circ$.

4. $\angle EFG$ and $\angle HFG$ are complementary. By the definition of complementary, $m\angle EFG + m\angle HFG = 90^\circ$. So, $m\angle HFG = 90^\circ - m\angle EFG$. By substitution, $m\angle HFG = 90^\circ - 20^\circ = 70^\circ$.

a. a pair of vertical angles - $\angle INJ$ and $\angle LNM$ OR $\angle INM$ and $\angle LNJ$

b. one linear pair of angles – multiple possible answers - $\angle INK$ and $\angle KNL$ OR $\angle INM$ and $\angle LNM$.

5. c. two complementary angles - $\angle INJ$ and $\angle JNK$

d. two supplementary angles - $\angle INK$ and $\angle KNL$ OR $\angle INM$ and $\angle LNM$

6. a. Since $\angle INJ$ and $\angle JNK$ are complementary, $m\angle INJ + m\angle JNK = 90^\circ$. So, using substitution, $63^\circ + m\angle JNK = 90^\circ$. Solving for the angle gives $m\angle JNK = 27^\circ$.

b. Since $\angle INK$ and $\angle KNL$ are supplementary, $m\angle INK + m\angle KNL = 180^\circ$. So, using substitution, $90^\circ + m\angle KNL = 180^\circ$. Solving for the angle gives $m\angle KNL = 90^\circ$

c. $\angle INJ$ and $\angle MNL$ are vertical angles; therefore, they are congruent. By the definition of congruent angles, $m\angle INJ = m\angle MNL$. Since $m\angle INJ = 63^\circ$, then $m\angle MNL = 63^\circ$.

d. Since $\angle INJ$ and $\angle MNI$ are supplementary, $m\angle INJ + m\angle MNI = 180^\circ$. So, using substitution, $63^\circ + m\angle MNI = 180^\circ$. Solving for the angle gives $m\angle MNI = 117^\circ$

## Classifying Triangles

1. sides – scalene (three sides of different lengths), angles – acute (three angles less than 90°)

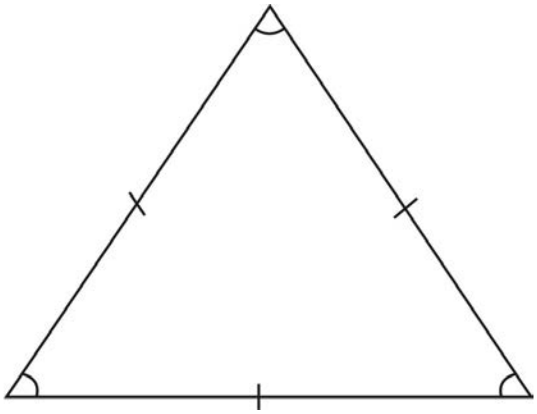
2. sides – equilateral (three sides of the same length) , angles – equiangular (equilateral triangles are also equiangular)

3. sides – isosceles (two side of the same length) , angles – right (a triangle with a right angle)

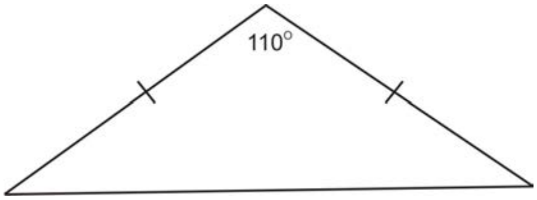
4. sides – scalene (three sides of different lengths), angles – not enough information (We cannot assume that any angle is 90°.)

5. sides – scalene (three sides of different lengths), angles – obtuse (a triangle with one obtuse angle)

6. Since the triangle is equiangular, it must also be equilateral, which means all the sides are congruent.



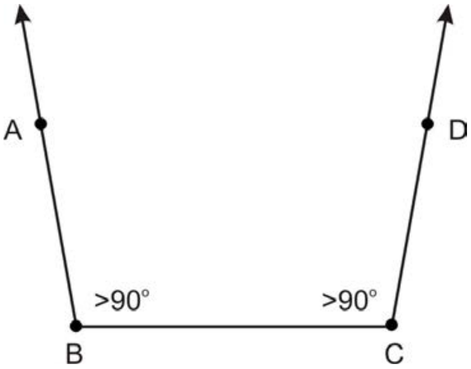
7.



8. This statement is true because a triangle can have a right angle, and have three different length sides (example a right triangle with sides $3, 4$, and 5 units).

9. This statement is false because the interior angles of a triangle must add up to 180°. By definition, obtuse angles have measures $> 90°$, so two obtuse angles would be $> 180°$. (And that does not even include the third angle!)

10. An obtuse triangle can only have one obtuse angle.



## Classifying Polygons

1. concave (because it caves in) and pentagon (because it has 5 sides)

2. convex (because it does not cave in) and octagon (because it has 8 sides)

3. convex (because it does not cave in) and 17−gon or heptadecagon (because it has 17 sides)

4. convex (because it does not cave in) and decagon (because it has 10 sides)

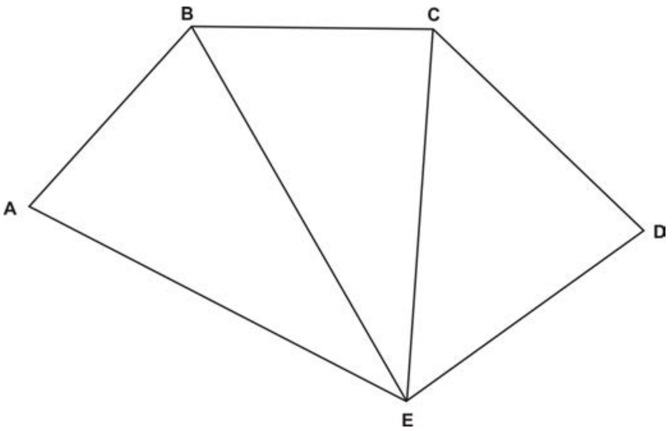
5. concave (because it caves in) and quadrilateral (because it has 4 sides)

6. A. This is not a polygon because two of the sides to not meet at a vertex.

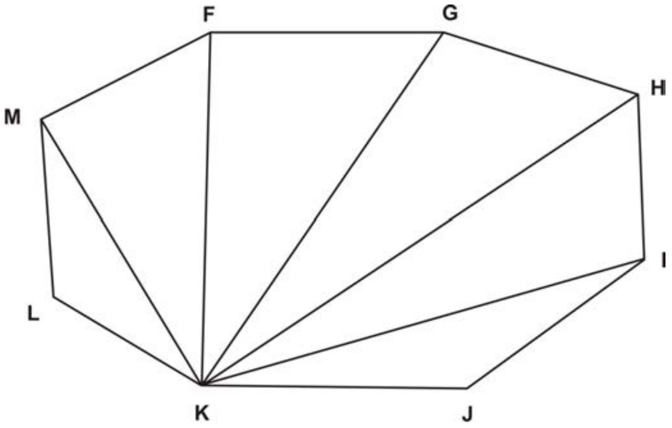
B. This is not a polygon because all the sides are not line segments (it has a curve).

C. This is not a polygon because it is not closed.

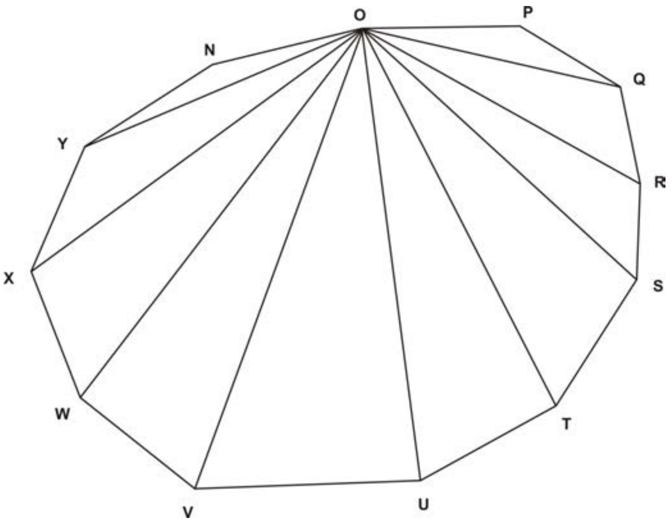
7. You can draw 2 diagonals from 1 vertex.



8. You can draw 5 diagonals from 1 vertex.



9. You can draw 9 diagonals from 1 vertex.



10. In general, you can use the following chart (or create your own).

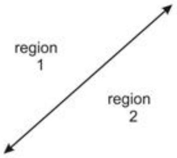
Table 1.1

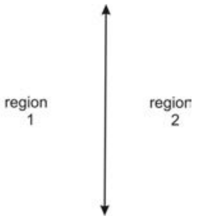
| Number of Sides | Diagonals from One Vertex | Process Sides $-3 =$ diagonals |
|---|---|---|
| 3 | 0 | $(3) - 3 = 0$ |
| 4 | 1 | $(4) - 3 = 1$ |
| 5 | 2 | $(5) - 3 = 2$ |
| 6 | 3 | $(6) - 3 = 3$ |
| 7 | 4 | $(7) - 3 = 4$ |
| 8 | 5 | $(8) - 3 = 5$ |
| 9 | 6 | $(9) - 3 = 6$ |
| 10 | 7 | $(10) - 3 = 7$ |
| 11 | 8 | $(11) - 3 = 8$ |
| 12 | 9 | $(12) - 3 = 9$ |
| ... | ... | |
| $n$ | $n - 3$ | $(n) - 3$ |

For any number of sides ($n$), the number of diagonals from one vertex is ($n - 3$).

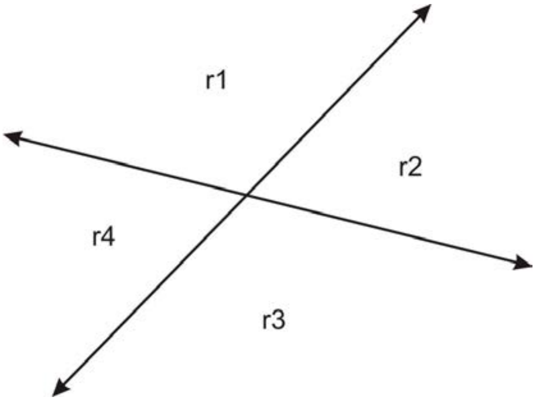
## Problem Solving in Geometry

1. If one line is drawn in a plane, the plane is divided into 2 regions (either to the left and right, or to the top and bottom).

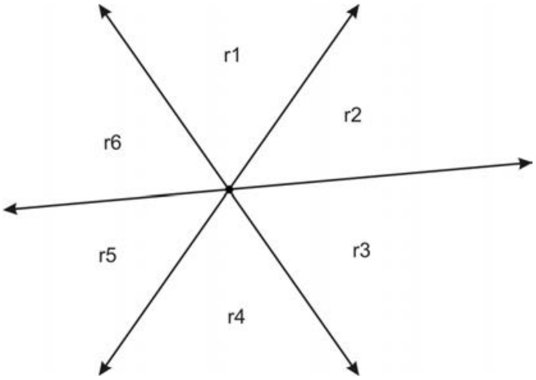




2. If two lines intersect in a plane, 4 regions are created.



3. If three lines intersect in a plane, 6 regions are created.



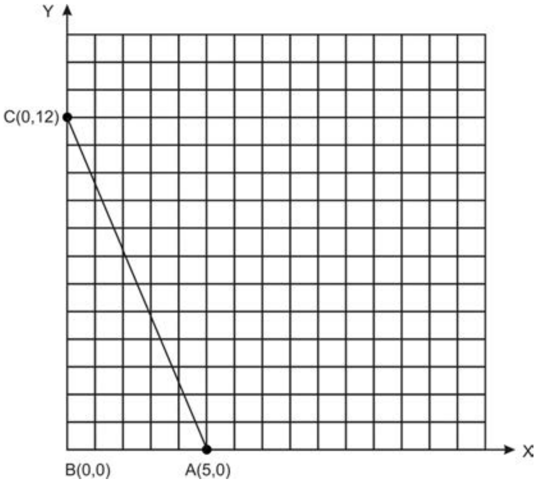
4. The table below will help you understand the relationship between the number of coplanar lines intersecting at one point and the number of regions created.

Table 1.2

| No. of coplanar lines intersecting at one point | Number of regions created | Process |
|---|---|---|
| 1 | 2 | $2(1) = 2$ |
| 2 | 4 | $2(2) = 4$ |
| 3 | 6 | $2(3) = 6$ |
| 4 | 8 | $2(4) = 8$ |
| 5 | 10 | $2(5) = 10$ |
| 6 | 12 | $2(6) = 12$ |
| 7 | 14 | $2(7) = 14$ |

5. From the table in item 4, the process is to multiply the number of coplanar lines ($n$) times 2. So, to determine the number of regions created by coplanar lines intersecting at the same point, the formula is $2n$.

6. a.



b. The coordinates of Cindy's house are $(0, 12)$, and the coordinates of Mari's house are $(5, 0)$.

c. The distance formula is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. If we assign the coordinates of Cindy's house (point $C$) at the first point, then $x_1 = 0$ and $y_1 = 12$. Then the coordinates of Mari's house (point $M$) as the second point, that makes $x_2 = 5$ and $y_2 = 0$. Substituting those values into the distance formula gives us $\sqrt{(5 - 0)^2 + (0 - 12)^2}$ which simplifies to $\sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. The distance between Cindy's house and Mari's house is 13 miles.

7. There are many ways to solve this problem, but some methods are beyond our mathematical grasp. As you continue learning more mathematics concepts, you will learn more efficient techniques. One method is below. You make have used a different method, but should still come up with the same result.

8. While the mathematics in number 7 is sound, it does not take into account other factors like obstacles that may make a certain path impassible. That might affect the path a runner took. In addition, most people would run slower with a full bucket of water than with an empty bucket, so the runner may wish to minimize the distance he/she has to run with a full bucket.

# Chapter 2

# TE Reasoning and Proof - Solution Key

## 2.1 Reasoning and Proof

### Inductive Reasoning

1. Considering how many dots are in the first 3 figures,

Figure 1 (3); Figure 2 (5); Figure 3 (7)

We can see that the pattern starts with 3 dots and increases by two each time, so Figure 4 will have 9 dots in it.

2. Considering how many dots are in the first 3 figures, the number of dots in the next pattern is dependent on the previous figure. In figure 2, there are 2 rows and 3 columns. In figure 3, there are 3 rows and 4 columns. The pattern is the number of columns in the previous figure will be the number of columns in the next figure. Then the number of rows will increase by one. Therefore, figure 4 will have 4 rows and 5 columns, for a total of $4 * 5 = 20$ dots.

3. In the 3 given figures, the pattern is to add 4 dots to the previous figure. Figure 1 has 1 dot, Figure 2 has 5 dots, and Figure 3 has 9 dots. It follows that Figure 4 will have $9 + 4 = 13$ dots.

4. In the sequence $5, 8, 11, 14$, the pattern is to add three to the previous number, so the next number is $14 + 3 = 17$.

5. In the sequence $3, 6, 11, 18$, the pattern is to add the next odd number to the previous number.

$$\begin{aligned} 3 + (3) &= 6 \\ 6 + (5) &= 11 \\ 11 + (7) &= 18 \end{aligned}$$

So, the next number will be $18 + (9) = 27$. Continuing the pattern, $27 + (11) = 38, 38 + (13) = 51, 51 + (15) = 66, 66 + (17) = 83, 83 + (19) = 102$.

6. The pattern in the table is the same as the pattern in item 5, where you add the next odd number to the

previous entry, so when $n = 6, t = 35 + 13 = 48$.

7. When $t = 99$, $n$ will be 9. We can determine the pattern in $t = n^2 + 2n$, so solving $99 = n^2 + 2n$ using the quadratic formula gives $n = 9$.

8. 145 is not a possible value for $t$. The first 12 values for $t$ are $3, 8, 15, 24, 35, 48, 63, 80, 99, 120, 143, 168$. If you substitute 145 in for $t$ into the equation $t = n^2 + 2n$, $n$ will be a decimal.

9. Consider the value $n = 1$. $n^2 = 1^2 = 1$

10. The number 2 is an even number and the only even prime. (Does that make it odd?)

11. If the points $A, B$, and $C$ are noncollinear, then the values may not hold true.

## Conditional Statements

1. hypothesis: 2 divides evenly into $x$

conclusion: $x$ is an even number

2. hypothesis: a triangle has three congruent sides

conclusion: it is an equilateral triangle

3. hypothesis: a triangle is equiangular

conclusion: it is equilateral

4. The converse is "If $x$ is an even number, then 2 divides into $x$." This statement is true.

5. The inverse is "If a triangle does not have three congruent sides, then is it not an equilateral triangle." This statement is true.

6. The contrapositive is "If a triangle is not equilateral, then it is not equiangular." This statement is true.

7. The statement is "If $b$ is the midpoint of $\overline{AC}$, then $AB = 5$ and $BC = 5$." This statement is false because $AB$ and $BC$ can be equal measures of any length.

8. The inverse of the inverse is the original statement, "If $p$ then $q$".

9. The converse of the inverse is the contrapositive.

10. The inverse of the converse is the contrapositive.

11. $p$ in words: a U.S. citizen can vote

$q$ in words: he or she is 18 or more years old

Is $p \rightarrow q$ true? yes

Is $q \rightarrow p$ true? yes

Is $p \leftrightarrow q$ true? yes

12. $p$ in words: a whole number is prime

$q$ in words: it is an odd number

Is $p \rightarrow q$ true? false

Is $q \rightarrow p$ true? false (consider the number 1)

Is $p \leftrightarrow q$ true? false (consider the number 2)

13. $p$ in words: points are collinear

$q$ in words: there is a line that contains the points

Is $p \rightarrow q$ true? true

Is $q \rightarrow p$ true? true

Is $p \leftrightarrow q$ true? true

14. $p$ in words: $x + y = 17$

$q$ in words: $x = 8$ and $y = 9$

Is $p \rightarrow q$ true? false (consider $x = 3$ and $y = 14$)

Is $q \rightarrow p$ true? true

Is $p \leftrightarrow q$ true? false

## Deductive Reasoning

1. The third sentence does not need to be true. One could be a smart person and not vote for Jane.

2. The third sentence does not need to be true. Maria may drive after any person.

3. The third sentence must be true because one cannot have an equilateral triangle that was not also equiangular.

4. If North wins, then East loses.

5. We know if $x = 6$, then $y > 7$ and $x > 3$.

6.

| $p$ | $\sim p$ | $p \wedge \sim p$ |
|---|---|---|
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |

7.

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ |

8.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $F$ |
| $F$ | $T$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $T$ | $T$ |

9.

| $p$ | $q$ | $\sim q$ | $q \vee \sim q$ | $p \wedge (q \vee \sim q)$ |
|---|---|---|---|---|
| $T$ | $T$ | $F$ | $T$ | $T$ |
| $T$ | $F$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ | $T$ | $F$ |

10. This statement $p \vee q \vee r$ is always true except when all values are false.

11. The only values that make this statement true are $x > -2$.

12. There are no values that make this statement true.

## Algebraic Properties

1. If $x = 3$, then $3 = x$.

2. If $x + 2 = 9$, then $x + 2 = 9$.

3. If $y = 12$ and $x = y$, then $x = 12$.

4. If $x + y = y + z$, then $y + z = x + y$.

5. If $x + y = y + z$, then $x + y = y + z$.

6. If $x = y - 7$ and $x = z + 4$, then $y - 7 = z + 4$.

7. By the transitive property of equality, if $a = b$ and $b = c$, then $a = c$. Also by the transitive property of equality, of $c = d$ and $d = e$, then $c = e$. Finally, if $a = c$ and $c = e$, then $a = e$.

8. This statement is not symmetric. If $X$ is the mother of $Y$, then $Y$ cannot be the mother of $X$.

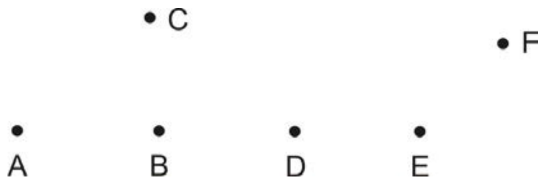
9. This statement is not symmetric. If John is the brother of Sally, then it does not follow that Sally is the brother of John.

10. This statement is not transitive. If $X$ is the mother of $Y$ and $Y$ is the mother of $Z$, then $X$ cannot be the mother of $Z$.

11. This statement is transitive. If Miguel is the brother of Juan and Juan is the brother of Isaac, then it follows that Miguel is the brother of Isaac.

12. We are given that $w, x, y$, and $z$ are real numbers and that $w = y$ and $x = z$. By the addition property of equality, we can add $x$ to both sides of the equation $w = y$ to get $w + x = y + x$. Then, since $x = z$, we can substitute $z$ in for $x$ on the right side of the equation to get $w + x = y + z$.

13.



$$AB = DE, BC = EF, AC \neq EF$$

## Diagrams

1. One right angle is $\angle BFG$.

2. $\overleftrightarrow{EH} \perp \overline{BF}$.

3. The statement $EG = FH$ is true. We are given that $EF = GH$. By the reflexive property of equality, we know $FG = FG$. We can use the Segment Addition postulate to write two different statements:

$EF + FG = EG$ and $FG + GH = FH$

By substitution, we get $FG + EF = FH$. We can set the left side of each equation equal to each other $EF + FG = FG + EF$ by the transitive property. Since the left sides of the equations are equal, by substitution, the right sides are equal, $EG = FH$.

4. In the diagram, we are given that $\overline{BC} \cong \overline{FG}$, and we are now given $\overline{BC} \times \overline{FG}$. Since we have a quadrilateral with one pair of opposite sides both parallel and congruent, then $BCGF$ is a rectangle.

5. $m\angle ABF = m\angle ABE + m\angle EBF$ because of the Angle Addition Postulate.

$m\angle DCG = m\angle DCH + m\angle HCG$ because of the Angle Addition Postulate.

6.

$$AB + BC = AC$$
$$BC + CD = BD$$

7.

Table 2.1

| Statement | Reason |
|---|---|
| 1. $\angle EBF \cong \angle HCG$ | 1. Given |
| 2. $m\angle EBF = m\angle HCG$ | 2. definition of congruent angles |
| 3. $\angle ABE \cong \angle DCH$ | 3. Given |
| 4. $m\angle ABE = m\angle DCH$ | 4. definition of congruent angles |
| 5. $m\angle EBF + m\angle ABE = m\angle ABF$<br>$m\angle HCG + m\angle DCH = m\angle DCG$ | 5. Angle Addition Postulate |
| 6. $m\angle EBF + m\angle ABE = m\angle DCG$ | 6. Substitution |
| 7. $m\angle ABF = m\angle DCG$ | 7. Substitution |
| 8. $\angle ABF \cong \angle DCG$ | 8. definition of congruent angles |

8.

Table 2.2

| Statement | Reason |
|---|---|
| 1. $AB = CD$ | 1. Given |
| 2. $AB + BC = AC$<br>$CD + BC = BD$ | 2. Segment Addition Postulate |
| 3. $CD + BC = AC$ | 3. Substitution |

Table 2.2: (continued)

| Statement | Reason |
|---|---|
| 4. $\overline{BC} \cong \overline{BC}$ | 4. reflexive property of congruence |
| 5. $BC = BC$ | 5. definition of congruent segments |
| 6. $AC = BD$ | 6. Substitution |

9. Two railroad tracks model parallel lines.

10. A floor and a ceiling model two parallel planes.

11. Depending on which lines one chooses, the model may be of parallel lines or perpendicular lines.

12. The referee's arms model parallel lines.

13. The letter "$L$" represents perpendicular segments.

14. The spine models perpendicular planes (with the book closed).

## Two-Column Proof

1. No, we do not have enough information to show this.

2. No, we do not have enough information to show this.

3. Yes, this information is given.

4. No, we do not have enough information to show this.

5. Yes, we are given that $\angle DAB$ is a right angle, so by the definition of perpendicular lines, we could show $\overline{AB} \perp \overline{AD}$.

6. No, we do not have enough information to show this.

7. No, we do not have enough information to show this.

8. No, we do not have enough information to show this.

9. No, we do not have enough information to show this.

10. No, we do not have enough information to show this.

11. There are 6 segments ($\overline{WX}, \overline{WY}, \overline{WZ}, \overline{XY}, \overline{XZ}, \overline{YZ}$).

12. Since $X$ is the midpoint of $\overline{WZ}$, then $WX = XZ = 6$. Since $Y$ is the midpoint of $XY = YZ = 3$. The, by the Segment Addition Postulate $WX + XY = WY$. By substitution, $WY = 6 + 3 = 9$.

13. Since $X$ is the midpoint of $\overline{WZ}$, then $WX = XZ = 6$.

14. By the symmetric property, $ZW = WZ = 12$.

15.

Table 2.3

| Statement | Reason |
|---|---|
| 1. $\angle DAB$ is a right angle | 1. Given |

Table 2.3: (continued)

| Statement | Reason |
|---|---|
| 2. $m\angle DAB = 90^\circ$ | 2. definition of a right angle |
| 3. $m\angle DAC + m\angle CAB = m\angle DAB$ | 3. Angle Addition Postulate |
| 4. $\overrightarrow{AC}$ bisects $\angle DAB$ | 4. Given |
| 5. $m\angle DAC = m\angle BAC$ | 5. definition of angle bisector |
| 6. $m\angle BAC + m\angle BAC = 90^\circ$ | 6. Substitution |
| 7. $2(m\angle BAC) = 90^\circ$ | 7. Simplify |
| 8. $m\angle BAC = 45^\circ$ | 8. division property of equality |

## Segment and Angle Congruence Theorems

1.

Table 2.4

| Statement | Reason |
|---|---|
| 1. $\angle A$ | 1. Given |
| 2. $m\angle A = m\angle A$ | 2. reflexive property of equality |
| 3. $\angle A \cong \angle A$ | 3. definition of congruent angles |

2.

Table 2.5

| Statement | Reason |
|---|---|
| 1. $\angle A \cong \angle B$ | 1. Given |
| 2. $m\angle A = m\angle B$ | 2. definition of congruent angles |
| 3. $m\angle B = m\angle A$ | 3. symmetric property of equality |
| 4. $\angle B \cong \angle A$ | 4. definition of congruent angles |

3.

Table 2.6

| Statement | Reason |
|---|---|
| 1. $\angle A \cong \angle B, \angle B \cong \angle C$ | 1. Given |
| 2. $m\angle A = m\angle B, m\angle B = m\angle C$ | 2. definition of congruent angles |
| 3. $m\angle A = m\angle C$ | 3. transitive property of equality |
| 4. $\angle A \cong \angle C$ | 4. definition of congruent angles |

4. The statement is true.

Table 2.7

| Statement | Reason |
|---|---|
| 1. $\angle A \cong \angle B, \angle C \cong \angle D$ | 1. Given |
| 2. $m\angle A = m\angle B, m\angle C = m\angle D$ | 2. definition of congruent angles |
| 3. $m\angle A + m\angle C = m\angle B + m\angle C$ | 3. addition property of equality |
| 4. $m\angle A + m\angle C = m\angle B = m\angle D$ | 4. Substitution |

5.

Table 2.8

| Statement | Reason |
|---|---|
| 1. $A, B, C$, and $D$ are collinear.<br>$\overline{AB} \cong \overline{CD}$ | 1. Given |
| 2. $AB = CD$ | 2. definition of congruent segments |
| 3. $AB + BC = AC, BC + CD = BD$ | 3. Segment Addition Postulate |
| 4. $CD + BC = AC$ | 4. Substitution |
| 5. $\overline{BC} \cong \overline{BC}$ | 5. reflexive property of congruence |
| 6. $BC = BC$ | 6. definition of congruent segments |
| 7. $AC = BD$ | 7. Substitution |
| 8. $\overline{AC} \cong \overline{BD}$ | 8. definition of congruent segments |

6. This statement is true. Since we are given $\overrightarrow{QS}$ is in the interior of $\angle PQR$, then we know by the Angle Addition Postulate that $m\angle PQS + m\angle SQR = m\angle PQR$. Likewise, since we are given $\overrightarrow{QT}$ is in the interior of $\angle PQS$, then we know by the Angle Addition Postulate that $m\angle PQT + m\angle TQS = m\angle PQS$. Therefore, we can say $(m\angle PQT + m\angle TQS) + m\angle SQR = m\angle PQR$ using substitution. Then by associative property of equality, $m\angle PQT + (m\angle TQS + m\angle SQR) = m\angle PQR$. We know now that $m\angle PQT + m\angle TQR = m\angle PQR$. Since the sum of the two small angles equals the measure of the larger angle, $\overrightarrow{QT}$ is in the interior of $\angle PQR$.

## Proofs About Angle Pairs

1. $m\angle 2 = 120^\circ$ by the Linear Pair Postulate.

2. $m\angle 3 = 60^\circ$ by the Vertical Angles Theorem.

3. $m\angle 4 = 120^\circ$ by the Linear Pair Postulate.

4.

Table 2.9

| Statement | Reason |
|---|---|
| 1. $\overline{AE} \perp \overline{EC}$ and $\overline{BE} \perp \overline{ED}$ | A. Given |
| 2. $\angle AEC$ and $\angle BED$ are right angles | B. definition of perpendicular lines |
| 3. $m\angle AEC = m\angle 1 + m\angle 2$ and $m\angle BECD = m\angle 2 + m\angle 3$ | C. Angle Addition Postulate |
| 4. $m\angle AEC = m\angle BED = 90^\circ$ | D. definition of right angles |

Table 2.9: (continued)

| Statement | Reason |
|---|---|
| 5. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 = 90^\circ$ | E. Substitution |
| 6. $\angle 1$ and $\angle 2$ are complimentary, $\angle 2$ and $\angle 3$ are complimentary | F. definition of complementary angles |
| 7. $\angle 1 \cong \angle 3$ | G. Compliments of same angle are congruent |

5.

a. No

b. Yes (Vertical Angles Theorem)

c. No

6. We are given that $\angle 2 \cong \angle 3$, so we know $m\angle 2 = m\angle 3$ by the definition of congruent angles. Also, we know the dashed line is perpendicular to the mirror, so $\angle 1$ and $\angle 2$ are complimentary, $\angle 3$ and $\angle 4$ are complimentary. By the definition of complimentary angles, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 = 90^\circ$. Then we can substitute to get $m\angle 1 + m\angle 3 = m\angle 3 + m\angle 4$. Finally, by the subtraction property of equality, we get $m\angle 1 = m\angle 4$. Therefore, $\angle 1 \cong \angle 4$ (the angle of incidence is congruent to the angle of reflection.)

# Chapter 3

# TE Parallel and Perpendicular Lines - Solution Key

## 3.1 Parallel and Perpendicular Lines

### Lines and Angles

1. Since the lines do not intersect and do not lie in the same plane, they are skew. It is difficult to see this on the picture.

2. The Parallel Postulate states that you can only draw one line that will be parallel to a given line ($m$) through a given point ($E$) that is not on the given line.

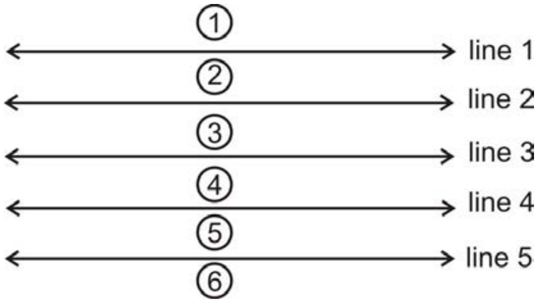
3. C. The definition of skew lines are noncoplanar lines that do not intersect.

4. No. Pyramids have sides that intersect. Even if the sides here do not intersect, the lines containing the sides eventually would.

5. The perpendicular postulate is similar to the Parallel Postulate. There is only one line perpendicular to the given line ($l$) through the given point ($M$).

6. A. Parallel lines are coplanar lines that do not intersect.

7.



The plane is divided into 6 regions.

8. If you draw $n$ parallel lines in a plane, you divide the plane into $n + 1$ regions.

9. Angles 1 and 5 are corresponding angles. Corresponding angles are in the same relative position. (In this example, above their respective lines, and to the left of the transversal.)

10. Angles 7 and 8 are a linear pair. The definition of linear pair is two adjacent angles whose non-common sides form a straight line.

## Parallel Lines and Transversals

1. $\angle AFG$ and $\angle CGH$ are corresponding angles. Corresponding angles are in the same relative position. (In this example, to the right of their respective lines, and below the transversal.)

2. $\angle BFG$ and $\angle DGF$ are supplementary. They are same-side interior angles, and it could be easily shown that their sum is 180°.

3. $\angle FGD$ and $\angle AFG$ are alternate interior angles. They are between the two parallel lines and on opposite sides of the transversal.

4. $\angle AFE$ and $CGH$ are supplementary. They are same side-exterior angles, thus they have a sum of 180°

5. Since lines $l$ and $m$ are parallel, corresponding angles are congruent. Therefore, $\gamma = 73°$.

6. The angle that measures 73° and $\beta$ form a linear pair. According to the Linear Pair Postulate, linear pairs are supplementary. So, $\beta + 73° = 180°$. By subtracting 73 from both sides, one gets $\beta = 107°$.

7. One can derive at $\theta$ in multiple ways. Since we have already calculated $\beta$ to be 107°, we can use that $\beta$ and $\theta$ are corresponding angles. Corresponding angles of parallel lines cut by a transversal are congruent. By the definition of congruent angles, $\theta = \beta = 107°$.

8. Since Jimenez Ave and Ella Street are parallel, the angle corresponding to the angle measuring 115° are congruent, which means they both measure 115°. Then that angle and $\angle 1$ form a linear pair. By the Linear Pair Postulate, they are supplementary. So, $m\angle 1 + 115° = 180°$ which means that $m\angle 1 = 65°$.

9. $\angle 1$ and $\angle 2$ form a linear pair; therefore, $m\angle 2 + 115° = 180°$ which means that $m\angle 2 = 65°$. You can check that by noticing that $\angle 1$ and $\angle 2$ are corresponding angles and therefore should be the same measure.

10. $\angle 1$ and $\angle 3$ form a linear pair; therefore, $m\angle 3 + 65° = 180°$ which means that $m\angle 3 = 115°$.

11.

Table 3.1

| Statement | Reason |
|---|---|
| 1.r ∥ s | 1. Given |
| 2. $\angle 1$ and $\angle 3$ are corresponding angles | 2. definition of corresponding angles |
| 3. $\angle 1 \cong \angle 3$ | 3. corresponding Angles Postulate |
| 4. $m\angle 1 = m\angle 3$ | 4. definition of congruent angles |
| 5. $\angle 2$ and $\angle 3$ form a linear pair | 5. definition of linear pair |
| 6. $m\angle 2 + m\angle 3 = 180°$ | 6. Linear Pair Postulate |
| 7. $m\angle 2 + m\angle 1 = 180°$ | 7. Substitution |
| 8. $\angle 1$ and $\angle 2$ are supplementary angles | 8. definition of supplementary angles |

## Proving Lines Parallel

1. $\overleftrightarrow{AF}$ and $\overleftrightarrow{CG}$ are parallel because of the Alternate Interior Angles Converse.

2. No, lines 1 and 2 are not parallel. For the lines to be parallel, alternate exterior angles must be congruent.

3. $\overleftrightarrow{MN}$ and $\overleftrightarrow{OP}$ are not parallel because alternate interior angles are not congruent.

4. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel because of the Corresponding Angles Converse.

5. $\overleftrightarrow{OR}$ and $\overleftrightarrow{LN}$ are parallel because $m\angle MSR + m\angle LTP = 180^\circ$, which means they are supplementary. If alternate exterior angles are supplementary, then the lines are parallel (Alternate Exterior Angles Converse).

6. $a = 50^\circ$ since it forms a linear pair with the angle that measures $130^\circ$.

7. Since we are given that $m \parallel n$, we know $b = h$ by the Corresponding Angles Postulate. If we can find $h$, then we will know $b$. The angle with a measure of $h$ and the angle with a measure of $140^\circ$ form a linear pair. Therefore, by the Linear Pair Postulate, $h + 140^\circ = 180$. Solving for $h$ gives us $h = 40^\circ$, so $b = 40^\circ$.

8. The angles with measures of $h$ and $c$ form a linear pair, so by the Linear Pair Postulate, $h + c = 180^\circ$. Substituting the value for $h$ we derived from # 7, the equation becomes $40^\circ + c = 180^\circ$, which means $c = 140^\circ$. We could verify this by understanding that the angle that measures $140^\circ$ in the original diagram and $c$ are vertical angles. According to the Vertical Angles Theorem, vertical angles are congruent. By the definition of congruent angles, $c = 140^\circ$.

9. To solve for $d$, we need to see that $a = d$ because the angles are corresponding angles of parallel lines cut by a transversal. Then, from #6, $a = 50^\circ$, so $d = 50^\circ$.

10. The right angle in the figure forms a linear pair with $e$. Thus, $90^\circ + e = 180^\circ$, which gives us $e = 90^\circ$.

11. The angles with measures of $b$ and $f$ form a linear pair, so $b + f = 180^\circ$. From #7, we know $b = 40^\circ$. Using substitution, $40^\circ + f = 180^\circ$. Solving for $f$ gives $f = 140^\circ$.

12. Since the angle measuring $130^\circ$ and $g$ are vertical angles, we know by the Vertical Angles Theorem that they are congruent. By the definition of congruent angles, $g = 130^\circ$.

13. We have already found $h$ in #7, so $h = 40^\circ$.

## Slopes of Lines

1. The best term for the slope is zero. Picking any two points on the line, say $(0, 1.5)$ and $(3, 1.5)$, and substituting hem into the slope formula $\left(m = \frac{y_2 - y_1}{x_2 - x_1}\right)$ gives us $m = \frac{1.5 - 1.5}{3 - 0} = \frac{0}{3} = 0$. More intuitively, consider walking on a road that is flat. It has zero slope because it is not going up or going down.

2. This line has a negative slope; consider walking down a hill. We can calculate the slope by picking points on the line, say $(-1, 2)$ and $(3, 0)$, and substituting them into the slope formula $\left(m = \frac{y_2 - y_1}{x_2 - x_1}\right)$ giving us $m = \frac{0 - 2}{3 - (-1)} = \frac{-2}{4} = -\frac{1}{2}$.

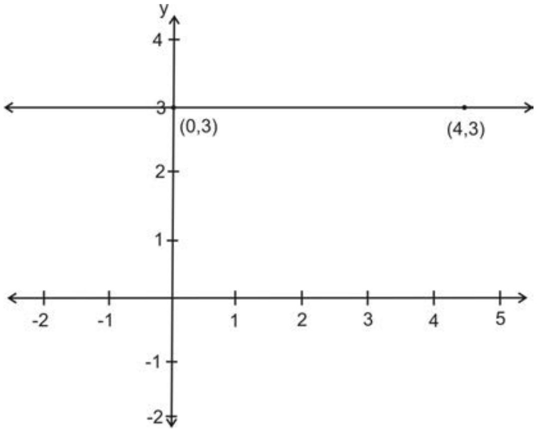
3. Using the slope formula, we can substitute the given points from the line, giving us $m = \frac{5 - 2}{1 - 0} = \frac{3}{1} = 3$.

4. First, we need to recall that parallel lines have the same slope. Therefore, if we calculate the slope of the given line, we will know the slope of any line that should be parallel to it. To find the slope of the given line, we need to pick 2 points on it. For this item, we will select points $(0, 0)$ because it is easy to use in calculations! and $(4, 1)$. Substituting these into the slope formula gives us $m = \frac{1 - 0}{4 - 0} = \frac{1}{4}$. Now we know that any line parallel to the given line must have a slope of $\frac{1}{4}$.

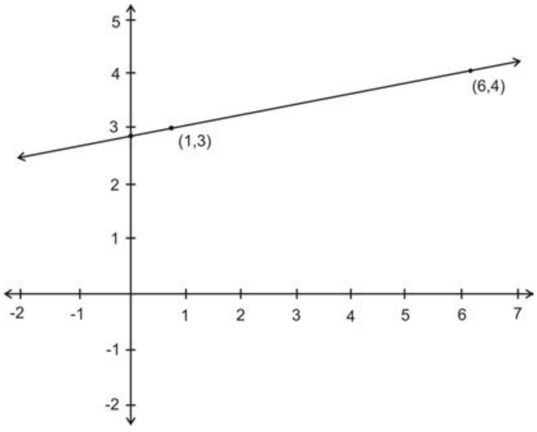
5. The slope of this line is undefined. Any vertical line has this slope. Consider falling off a vertical cliff. While the outcome is clearly defined, the slope of the cliff is not! We can confirm this slope by picking two points on the line and plugging them to the slope formula. We will pick $(3,0)$ and $(3,3)$. Using the slope formula gives us $m = \frac{3-0}{3-0} = \frac{3}{0}$. Since we cannot divide by zero, the fraction is undefined; therefore, so is the slope.

6. We can calculate the slope of this line by picking two points on the line and substituting them into the slope formula. We want to pick points that will make for easy calculations, so we will use the $x-$ and $y-$intercepts, which are $(6,0)$ and $(0,3)$, respectively. Using the slope formula gives us $m = \frac{3-0}{0-6} = \frac{3}{-6} = -\frac{1}{2}$.

7. Remember that a line with zero slope will be flat. If we recall that slope $= \frac{\text{rise}}{\text{run}}$, and that a fraction equal to zero has the 0 on top, which would correspond to the rise, we can see that the line will not rise at all no matter what the run is.



8. Recall that slope $= \frac{\text{rise}}{\text{run}}$. Since the slope of our line is $\frac{1}{5}$ , rise $= 1$ and run $= 5$. That means every time the line goes up 1 place in the $y-$direction, it will go to the right 5 places in the $x-$direction. For this particular example, we can add to the existing point $(1,3)$. Since the line will go up 1 point, we add 1 to the existing $y-$coordinate, $3 + 1 = 4$. We can do the same with the $x-$coordinate, $1 + 5 = 6$. So, another point on the graph is $(6,4)$. We can plot this point, and draw our line through the points.



9. We must recall that perpendicular lines have slopes that are negative reciprocals of each other. So, we should calculate the slope of the given line first, and then we can determine the slope of the perpendicular line by flipping the slope and changing the sign of our result. The given line passes through points $(0,-3)$ and $(1,1)$, so $m = \frac{1-(-3)}{1-0} = \frac{4}{1} = 4$. Since the slope of the given line is $\frac{4}{1}$, the slope of any line perpendicular

to it (commonly denoted $m_\perp$) would be $m_\perp = -\frac{1}{4}$.

10. We will use the same technique as we did in #8. The line passes through $(2, 6)$. Since the line has a slope of $-2$, or $\frac{-2}{1}$, another point on the line would be $(2 + 1, 6 - 2) = (3, 4)$. Notice that since the top number is negative, we subtract from the original $y-$coordinate instead of adding. We can plot the new point, and draw our line through the points.

## Equations of Lines

1. Since parallel lines have the same slope, we must pick the equation with the same slope as the original, namely $-\frac{1}{4}$. Using this knowledge, the only equation wit the same slope is a. $y = -\frac{1}{4}\ x - \frac{1}{2}$.

2. To write the equation of the line, we need the slope ($m$) and the $y-$intercept ($b$). We can look at the graph and see the line crosses the $y-$axis at $(0, -1)$, so $b = -1$. We can use the point $(0, -1)$ and another point, say $(3, 1)$, to calculate the slope of the line. Using these points yields $m = \frac{1-(-1)}{3-0} = \frac{2}{3}$. Substituting these values into the equation $y = mx + b$ gives us the equation of the graph, $y = \frac{2}{3}\ x - 1$.

3. Parallel lines have equal slopes, so we need to first identify the slope of the given line, which is $m = \frac{3}{2}$. So, we are looking for an equation with the same slope. The answer is *c.* $y = \frac{3}{2}\ x - 12$.

4. We will use the same technique here as we did in #2. The line crosses the $y-$axis at $(0, 4)$, so $b = 4$. The line also passes through $(3, 3)$, so we can use these two points in the slope formula to give us $m = \frac{3-4}{3-0} = \frac{-1}{3} = \frac{1}{3}$. Substituting into $y = mx + b$ yields $y = -\frac{1}{3}\ x + 4$.

5. Since perpendicular lines have negative reciprocal slopes, we need to identify the slope of the given line first, which is $-\frac{4}{7}$. The slope of a line perpendicular to the given line would be $m_\perp = \frac{7}{4}$. The only equation with that slope is *c.* $y = \frac{7}{4}\ x$.

6. Using the same technique from #2 and #4, we identify the $y-$intercept as $(0, 2)$, therefore $b = 2$. Using the $x-$intercept $(-4, 0)$ as the other point, we can substitute into the slope formula to give us $m = \frac{0-2}{-4-0} = \frac{-2}{-4} = \frac{1}{2}$ . Thus, the equation of the line is $y = \frac{1}{2}\ x + 2$.

7. Following the technique from #5, we identify the slope of the given line as $m = \frac{5}{6}$. Any line perpendicular to the given line will have slope $m_\perp = -\frac{6}{5}$, so the solution is *b.* $y = -\frac{6}{5}\ x - 7$.

8. [**Note:the solution in the text is incorrect**]

We want our equation to be in the form $Ax + By = C$. So, our first step is to multiply both sides of the equation by the lowest common denominator, 21. $(21)(y) = \left(\frac{1}{7}\ x + \frac{2}{3}\right)(21) \Rightarrow 21y = 3x + 14$. Then we subtract $3x$ from both sides, giving us $21y - 3x = 3x + 14 - 3x \Rightarrow -3x + 21y = 14$ which is in standard form.

9. [**Note:the solution in the text is incorrect**]

We will follow the same technique as in #8 to get our equation in standard form. $(20)(y) = \left(\frac{6}{5}\ x + \frac{1}{4}\right)(20) \Rightarrow 20y = 24x + 5$ . Next, we subtract $24x$ from both sides, giving us $20y - 24x = 24x + 5 - 24x \Rightarrow -24x + 20y = 5$, which is the standard form.

10. [**Note:the solution in the text is incorrect**]

We will solve this the same way as with #8 and #9. First, multiplying through by the common denominator gives us $(24)(y) = \left(-\frac{2}{3}\ x + \frac{1}{8}\right)(24) \Rightarrow 24y = -16x + 3$. Adding $16x$ to both sides yields the final standard form of $24y + 16x = -16x + 3 + 16x \Rightarrow 16x + 24y = 3$.

## Perpendicular Lines

1. $\angle LMN$ and $\angle OMN$ form a linear pair, so they are supplementary. By the definition of supplementary, $m\angle LMN + m\angle OMN = 180^\circ$. Since we know $\angle OMN$ is a right angle from the diagram, we $m\angle OMN = 90^\circ$ by the definition of right angles. Substituting into the equation, we get $m\angle LMN + 90^\circ = 180^\circ$. Solving for $m\angle LMN$ yields $m\angle LMN = 90^\circ$.

2. By the definition of complementary, $m\angle B + m\angle E = 90^\circ$.Since $m\angle B = 8^\circ$, we get $8^\circ + m\angle E = 90^\circ \Rightarrow m\angle E = 82^\circ$.

3. In the diagram, the right angle and the angle with measure $x$ are vertical angles. By the Vertical Angles Theorem, these angles are congruent; therefore, $x = 90^\circ$.

4. Since $l_1$ and $l_2$ are parallel, $a = 40^\circ$ by the Corresponding Angles Postulate.

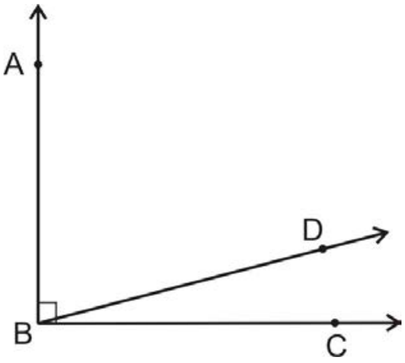
5. Angles $a$ and $b$ form a linear pair, so they are supplementary by the Linear Pair Postulate. Since $a = 40^\circ$ (from #4), we can find b by using $b = 180^\circ - 40^\circ = 140^\circ$.

6. Since $l_1$ and $l_2$ are parallel, $c$ and $b$ are equal. So, since $b = 140^\circ$ (from #5), $c = 140^\circ$ by the Corresponding Angles Postulate.

7. $d = 90^\circ$ because vertical angles are congruent, and the angle with $d$ is vertical angles with a right angle.

8. Since we know that $l_1$ and $l_2$ are parallel and $l_1$ and $l_3$ are perpendicular, it can be shown that $l_3$ and $l_2$ are perpendicular, therefore they intersect at a right angle, so $e = 90^\circ$

9.



$\angle ABD$ and $\angle DBC$ are adjacent and complementary.

10. Angles that are complimentary have a sum of $90^\circ$, and two angles that are congruent have the same measure. If $x$ is the measure of each angle, then $2x = 90$, which gives us $x = 45^\circ$.

11. Look at explanation for #1. Linear pairs are supplementary, so $m\angle PRS + m\angle SRT = 180^\circ$. We are given that $\angle PRS \cong \angle SRT$, so by the definition of congruent angles, $m\angle PRS = m\angle SRT$. Using substitution, we get $m\angle PRS + m\angle PRS = 180^\circ \Rightarrow 2(m\angle PRS) = 180^\circ \Rightarrow m\angle PRS = 90^\circ$. Since $m\angle PRS = 90^\circ$ , then $m\angle SRT = 90^\circ$.

## Perpendicular Transversals

1. Lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{GF}$ are parallel because of the Converse Theorem of Perpendicular Transversals.

2. $m\angle HFE = 90^\circ$ because it is a right angle. We know this because when lines intersect, they form 4 right angles.

3. We know from #1 that lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{GF}$ are parallel. From this, we can say $\angle JEL \cong \angle ECD$ because of the Corresponding Angles Postulate. By the definition of congruent angles, then, we know $m\angle JEL = \angle ECD = 35^\circ$. From the diagram, we also know that $\angle ECD$ and $\angle BCK$ are vertical angles. By the Vertical Angles Theorem, $\angle BCK \cong \angle ECD$, so $m\angle ECD = m\angle BCK = 35^\circ$.

4. Using results from #2 and #3 will help us solve for $m\angle KCI$. Since $\overleftrightarrow{BD}$ and $\overleftrightarrow{IH}$ are perpendicular, $\angle BCI$ is a right angle, thus $m\angle BCI = 90^\circ$. $\angle KCI$ and $\angle BCK$ are complementary, so $m\angle KCI + m\angle BCK = 90^\circ$. Substituting the result from #3 gives us $m\angle KCI + 35^\circ = 90^\circ \Rightarrow m\angle KCI = 55^\circ$.

5. Using results from #4 will help us solve for $m\angle BCE$. Since $\angle BCF$ is a right angle, $m\angle BCF = 90^\circ$. $\angle KCI$ and $\angle FCE$ are vertical angles; therefore, they are congruent. This lets us know $m\angle KCI = m\angle FCE = 55^\circ$. According to the Angle Addition Postulate, $m\angle BCF + m\angle FCE = m\angle BCE$. Substituting our values gives us $90^\circ + 55^\circ = m\angle BCE = 145^\circ$

6. [**Note: The solution in the text says A, implying multiple choices, but I did not see any**]

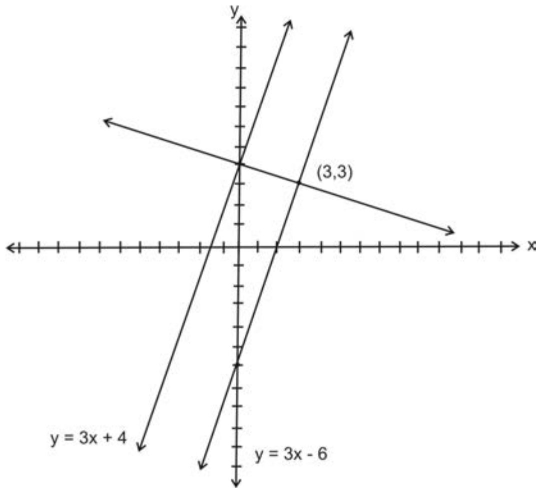
The distance between two lines is the perpendicular distance between the two lines. Since these two lines are horizontal lines, the perpendicular distance is just the difference in the $y-$coordinates along the same vertical line (example, on the line $x = 1$, the coordinates on the two lines are $(1, -1)$ and $(1, 6)$. Therefore the distance between the lines is $d = 6 - (-1) = 7$ units.

7. Following the same technique from #6, we can pick any two points on a horizontal line that intersects the two vertical lines given. The points we will use are $(3, 0)$ and $(8, 0)$. The distance is $d = 8 - 0 = 8$ units.

8. The slope of each line is $m = 3$. We know this because both equations are in slope-intercept form, so the coefficient of $x$ is the slope.

9. The slope of a line perpendicular to these lines must have a slope of $m_\perp = -\frac{1}{3}$.

10.

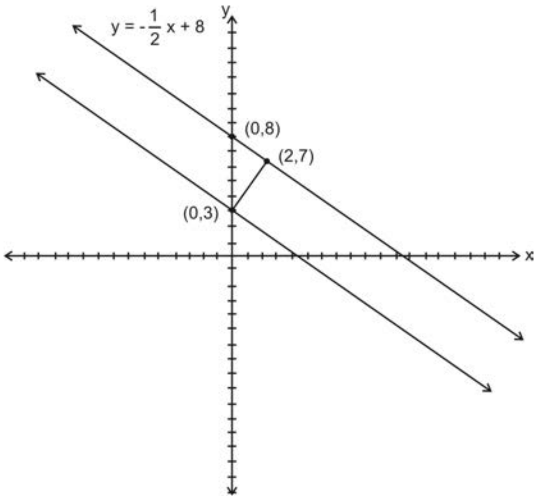


11. By looking at the graph, we can see the perpendicular line intersects $y = 3x - 6$ at $(3, 3)$.

12. $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(4-3)^2 + (0-3)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{10} \approx 3.16$

13. The slope of both lines is $m = -\frac{1}{2}$, so the slope of a line perpendicular them both is $m_\perp = 2$.

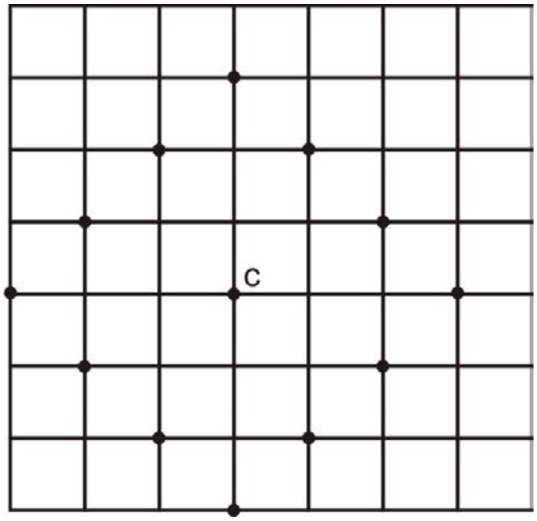
The graph shows the perpendicular line drawn through $(0, 3)$ in the first line and passes through $(1, 5)$. To get $(1, 5)$ we used the slope on the graph to move up 2 and to the right 1.



Alternatively, one could add to the original point $(0+1, 3+2) = (1,5)$. The perpendicular line will intersect the second parallel line at $(2,7)$. Now, we can calculate the length of the perpendicular distance between the parallel lines by using the points $(0,3)$ and $(2,7)$.

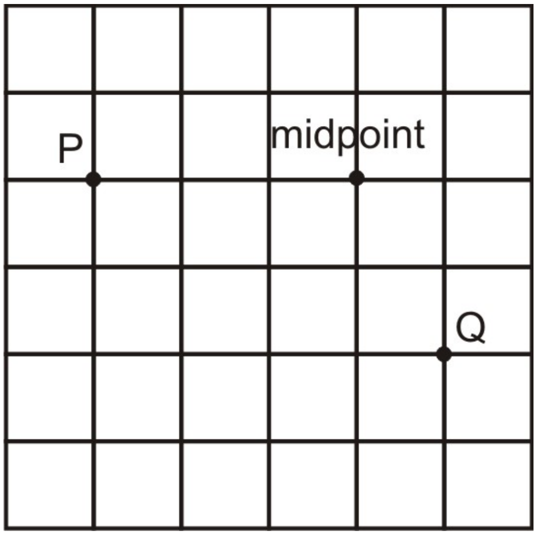
$$d = \sqrt{(2-0)^2 + (7-3)^2} = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5} \approx 4.47 \text{ units.}$$

## Non-Euclidean Geometry

1. The taxicab distance is 8 units (either 4 up then 4 right, or 4 right then 4 up).
2. 

3. The distance is 6 units; 4 units right and 2 units down.

4.



5. There are 3 midpoints.

6. The distance from $(3,5)$ and $(22,9)$ is 23 units.

7. Since the distance between the two points is odd, there is no midpoint.

8. There are several midpoints between $(2,5)$ and $(10,1)$. Any of the following would work: $(4,1),(5,2),(6,3),(7,4)$, or $(8,5)$.

9. There are 4 midpoints between the points. They are $(3,7),(4,8),(5,9)$, and $(6,10)$.

10. No. It is necessary to know the coordinates because if they have the same $x-$ or $y-$ coordinates, there would only be one midpoint, but there could be as many as six.

11. A chart describing the attributed may be a good way of seeing what they have in common and how they are different.

Table 3.2

| Taxicab | Euclidean |
|---|---|
| Distances are integers | Distances can be any real numbers |
| Could have multiple midpoints | Only one midpoint |
| Uses lines | Uses lines |
| Lines must be either horizontal or vertical | Lines can have any slope |
| Circles look like squares in Euclidean geometry | |

# Chapter 4

# TE Congruent Triangles - Solution Key

## 4.1 Congruent Triangles

### Triangle Sums

1. $\angle BAC$ and $\angle BAD$ form a linear pair. So, by the Linear Pair Postulate, $m\angle BAC + m\angle BAD = 180^\circ$. Substituting the given value in for $m\angle BAD$ gives us $m\angle BAC + 130^\circ = 180^\circ$. Solving for the angle gives us $m\angle BAC = 50^\circ$.

2. There are two ways to solve for $m\angle ABC$ in the triangle above.

(a) According to the Triangle Sum Theorem, all the angles in the triangle have a sum of 180°, or $m\angle ABC + m\angle BAC + \angle CAB = 180^\circ$. From #1, we know $m\angle BAC = 50^\circ$. So using the given value, $m\angle ABC + 50^\circ + 40^\circ = 180^\circ$. Solving for $m\angle ABC$ gives us $m\angle ABC = 90^\circ$.

(b) According to the Remote Exterior Angle Theorem, the measure of the exterior angle is equal to the sum of the remote interior angles, or $m\angle BAD = m\angle BAC + m\angle CAB$. Substituting the given values yields the equation $130^\circ = m\angle BAC + 40^\circ$. Solving for the angle gives us $90^\circ = m\angle BAC$.

3. $\angle VTS$ and $\angle VTU$ form a linear pair, so $m\angle VTS + m\angle VTU = 180^\circ$. We are given $m\angle VTS = 111^\circ$, so $m\angle VTU = 180^\circ - 111^\circ = 69^\circ$.

4. $\angle WVT$ and $\angle TVU$ form a linear pair, so $m\angle WVT + m\angle TVU = 180^\circ$. We know that $\angle WVT$, so by the definition of a right angle, $m\angle WVT = 90^\circ$, so $m\angle TVU = 180^\circ - 90^\circ = 90^\circ$.

5. From #4 and #5, $m\angle VTU = 69^\circ$ and $m\angle TVU = 90^\circ$. The Triangle Sum Theorem tells us that $m\angle VTU + m\angle TVU + m\angle TUV = 180^\circ$, so by substitution, $69^\circ + 90^\circ + m\angle TUV = 180^\circ$. Solving for the angle yields $m\angle TUV = 29^\circ$.

6. Since the angles have a sum of 90°, they are complementary.

7. By the Triangle Sum Theorem, $m\angle G + m\angle F + m\angle H = 180^\circ$. We also know that $m\angle H = m\angle F$ because $\angle H \cong \angle F$. So, by substituting the given values in, we get $40^\circ + m\angle F + m\angle F = 180^\circ$ or $(2m\angle F) = 140^\circ$. Solving for the angle gives us $m\angle F = 70^\circ$.

8. $a = 58^\circ$ because it forms a linear pair with the angle measuring 112°.

9. $b = 58°$ because same side interior angles of parallel lines cut by a transversal are supplementary.

10. $c = 25°$ because alternate interior angles of parallel lines cut by a transversal are congruent.

11. Since angle $c$ and angle $d$ form a linear pair, they are supplementary. From #10, we know $c = 25°$, then $d = 180° - 25° = 155°$.

12. We know by the Triangle Sum Theorem that the three angles in the interior of the triangle have a sum of 180°. We also know that the third angle at the top of the triangle is bisected, so we can say the measure of the angle is $2e$. Using this information gives us $2e + 58° + 25° = 180°$. Solving for $e$ gives $e = 97°$.

13. By the Remote Exterior Angle Theorem, we know that $e + a = f$. From #8, we know $a = 58°$ and from #12 we know $e = 97°$, so $f = 58° + 97° = 155°$.

14.

Table 4.1

| Statement | Reason |
|---|---|
| 1. $\triangle ABC$ | 1. Given |
| 2. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | 2. Triangle Sum Theorem |
| 3. $\angle 3$ and $\angle 4$ form a linear pair | 3. Definition of linear pair |
| 4. $m\angle 3 + m\angle 4 = 180°$ | 4. Linear Pair Postulate |
| 5. $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$ | 5. Substitution |
| 6. $m\angle 1 + m\angle 2 = m\angle 4$ | 6. Subtraction Property of Equality |

## Congruent Figures

1. There are multiple ways to write the congruence statement, but the vertices must line up. Example $\triangle PQR \cong \triangle NML$.

2. $\triangle BCD \cong \triangle YWX$

3. If two triangles are congruent, then the corresponding parts of those triangles are congruent (CPCTC). Since $\angle B$ corresponds to $\angle Y$ in their respective triangles, they must be congruent.

4. According to the Third Angles Theorem, if two angles of one triangle are congruent to two angles in another triangle, then the third angles are congruent.

5. For two figures to be congruent, all corresponding sides and angles must be congruent. $\overline{KL}$ corresponds to $\overline{WX}$, yet they are not congruent; therefore, the two triangles are not congruent.

6. You will need to measure two angles with a protractor. The third angle can then be found using the Triangle Sum Theorem.

7. $\angle FGH \cong \angle FGI$. We know this because the other two pairs of corresponding angles are congruent, so by the Third Angles Theorem, these angles must be congruent.

8. **Note: This item cannot be solved unless a measure is given.**

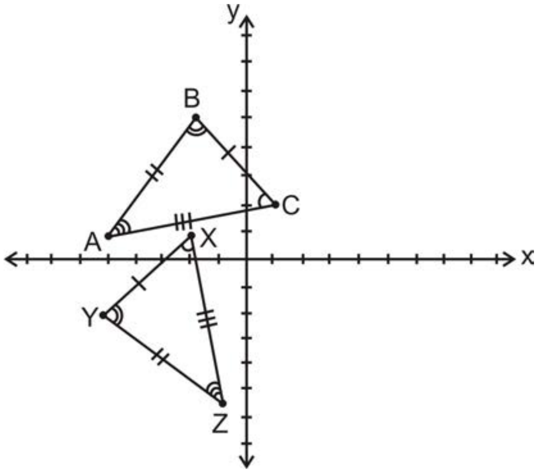
9. The reflexive property of congruence tells us that $\overline{FG} \cong \overline{FG}$.

10. $\triangle FGH \cong \triangle FGI$

## Triangle Congruence using SSS

1. We know $\angle P \cong \angle S, \angle Q \cong \angle T, \angle R \cong \angle U, \overline{PQ} \cong \overline{ST}, \overline{QR} \cong \overline{TU}, \overline{RP} \cong \overline{US}$.

2.



3. a. $AB = \sqrt{(1-5)^2 + (-5-(-2))^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

b. $BC = \sqrt{(2-5)^2 + (1-(-2))^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{2 \cdot 9} = 3\sqrt{2}$ units

c. $AC = \sqrt{(2-1)^2 + (1-(-5))^2} = \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$ units

4. a. $XY = \sqrt{(1-(-2))^2 + (-2-(-5))^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{2 \cdot 9} = 3\sqrt{2}$ units

b. $YZ = \sqrt{(-5-(-2))^2 + (-1-(-5))^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

c. $XZ = \sqrt{(1-(-5))^2 + (-2-(-1))^2} = \sqrt{(6)^2 + (-1)^2} = \sqrt{36+1} = \sqrt{37}$ units

5. $\triangle ABC \cong \triangle ZYX$ because $\overline{AB} \cong \overline{ZY}, \overline{BC} \cong \overline{YX}, \overline{AC} \cong \overline{ZX}$

6. $\triangle BCA \cong \triangle YXZ$ (as long as the vertices line up, the order does not matter).

7. Since all three sides of one triangle are congruent to the corresponding sides of the other triangle, then these triangles are congruent by SSS.

8. $\triangle TRI \cong \triangle CAB$ by SSS Congruence Postulate

9. Since the triangles are congruent, $\angle C \cong \angle T$ so $m\angle C = 39^\circ$.

10. Since we found $m\angle C = 39^\circ$ in #9, then we can use the Triangle Sum Theorem to find $m\angle A$. $m\angle A = 180^\circ - (39^\circ + 66^\circ) = 75^\circ$. Since $\angle A \cong \angle R,\ m\angle R = 75^\circ$.

## Triangle Congruence Using ASA and AAS

1. $\triangle PQR \cong \triangle BCA$

2. Since two angles and a non-included side of one triangle are congruent to two corresponding angles and a corresponding non-included side of another triangle, they are congruent by the AAS (Angle-Angle-Side) Triangle Congruence Postulate.

3. We know $\angle R \cong \angle A,\ \overline{QR} \cong \overline{CA}$, and $\overline{PQ} \cong \overline{BC}$.

4. A congruence statement is not possible.

5. The non-included sides do not correspond.

6. $\angle A \cong \angle D,\ \angle C \cong \angle F,\ \angle B \cong \angle E$

7. $\angle POC \cong \angle RAM$

8. Since two angles and a included side of one triangle are congruent to two corresponding angles and a corresponding included side of another triangle, they are congruent by the ASA (Angle-Side -Angle) Triangle Congruence Postulate.

9. $\angle P$ **is not congruent to** $\angle A$] We know $\angle P \cong \angle R$, $\overline{PC} \cong \overline{RM}$, and $\overline{PO} \cong \overline{RA}$.

10.

Table 4.2

| Statement | Reason |
|---|---|
| 1. $\angle L \cong \angle N$ | 1. Given |
| 2. $\angle P \cong \angle O$ | 2. **GIVEN** |
| 3. $\overline{LM} \cong \overline{MN}$ | 3. Given |
| 4. $\triangle LMP \cong \triangle NMO$ | 4. **AAS** triangle congruence postulate |
| 5. $\angle PML \cong \angle OMN$ | 5. CPCTC (corresponding parts of congruent triangles are congruent) |

11. When more than one angle is formed at a vertex, naming an angle at that vertex can be confusing, so it is necessary to name an angle at a common vertex with 3 letters. If there is only one angle formed at a vertex, it is acceptable to name it by just using the vertex.

## Proof Using SAS and HL

1. $\triangle RGT \cong \triangle NPU$

2. Since the hypotenuse and one leg of one triangle are congruent to the hypotenuse and a corresponding leg of another triangle, then the triangles are congruent by HL (hypotenuse-leg) Triangle Congruence Postulate.

3. $\overline{RG} \cong \overline{NP}$, $\angle R \cong \angle N$ and $\angle T \cong \angle U$.

4. $\triangle TAR \cong \triangle PIM$

5. These triangles are congruent by SAS (side-angle-side) Triangle Congruence Postulate because two sides and the included angle of one triangle are congruent to two corresponding sides and the included angle of another triangle.

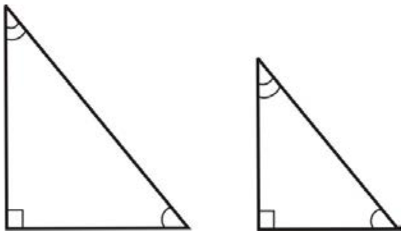
6. $\angle I \cong \angle A$, $\angle P \cong \angle T$, and $\overline{IP} \cong \overline{AT}$

7. It is not possible to write a congruence statement.

8. There is no theorem or postulate for SSA.

9. There are no other congruence statements that can be made.

10. If three angles of one triangle are congruent to another triangle, the sides may be different lengths.



11.

Table 4.3

| Statement | Reason |
|---|---|
| 1. $\overline{NO} \cong \overline{OQ}$<br>2. $\overline{MO} \cong \overline{PO}$<br>3. $\overline{MQ}$ and $\overline{NP}$ intersect at $O$.<br>4. $\angle MON$ and $\angle POQ$ are vertical angles<br>5. $\angle MON \cong \angle POQ$<br>6. $\triangle MON \cong \triangle POQ$<br>7. $\angle NMO \cong \angle PQO$ | 1. Given<br>2. Given<br>3. Given<br>4. definition of vertical angles<br>5. Vertical Angles Theorem<br>6. SAS<br>7. CPCTC |

## Using Congruent Triangles

1. Since $\overline{AB}$ is a horizontal line, it is not necessary to use the distance formula, so $AB = |-5-(-1)| = |-4| = 4$ units.

2. Since $\overline{BC}$ is a vertical line, it is not necessary to use the distance formula, so $BC = |3-(-2)| = |5| = 5$ units.

3. The $m\angle ABC = 90^\circ$. We know this because the two sides of the angle are perpendicular. We know they are perpendicular because vertical lines and horizontal lines are always perpendicular to each other.

4. One could use either SAS, SSS, or HL to show the triangles are congruent.

5. $AC = \sqrt{(3-(-2))^2 + (-5-(-1))^2} = \sqrt{(5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$. Once we show that $\triangle ABC \cong \triangle JKL$, then we will know $\overline{AC} \cong \overline{JL}$ by CPCTC. Then, by definition of congruent segments, $AC = JL$, which gives $JL = \sqrt{41}$.

6. $\triangle PAL \cong \triangle BUD$ by SAS Triangle Congruence Postulate.

7. $\triangle BIN \cong \triangle RAT$ by SSS Triangle Congruence Postulate.

8. There is not enough information to write a congruence statement.

9. Since Midtown is halfway between Uptown and Downtown, we can conclude that it is the midpoint of the segment from Uptown to Downtown. Also, there is a set of vertical angles in the diagram, and by the Vertical Angles Theorem, they are congruent. We are given that there are right angles in each triangle; therefore, by AAS, the triangles are congruent. Since the triangles are congruent, the segment from the Upper West Side to Uptown is congruent to the segment from Lower East Side to Downtown by CPCTC. This fact implies the distance from Lower East Side to Downtown is 1.5 km.

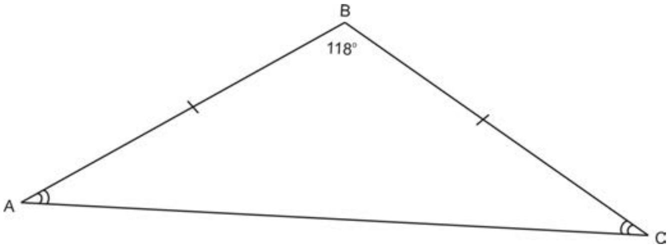
10.

Table 4.4

| Statement | Reason |
|---|---|
| 1. $R$ is the midpoint of $\overline{PN}$ | 1. Given |
| 2. $\overline{PR} \cong \overline{NR}$ | 2. definition of midpoint |
| 3. $\overline{PA} \parallel \overline{LN}$ | 3. Given |
| 4. $\angle A \cong \angle L$ | 4. Alternate Interior Angles Theorem |
| 5. $\angle PRA$ and $\angle NRL$ are vertical angles | 5. definition of vertical angles |
| 6. $\angle PRA \cong \angle NRL$ | 6. Vertical Angles Theorem |
| 7. $\triangle PRA \cong \triangle NRL$ | 7. AAS |
| 8. $\overline{PA} \cong \overline{LN}$ | 8. CPCTC |

## Isosceles and Equilateral Triangles

1.



2. If we let the measure of each base angle be represented by $x$, we have $118^\circ + x^\circ + x^\circ = 180^\circ$.

Simplifying gives us $2x + 118 = 180$.

Solving for $x$ gives the solution $x = 31$.

3. Since $\triangle QRS$ is isosceles and $\overline{RQ} \cong \overline{SQ}$, we know $m\angle R = m\angle S = 64^\circ$ By the triangle sum theorem, we know $m\angle Q + 64^\circ + 64^\circ = 180^\circ$ so, $m\angle Q = 52^\circ$

4. a. $m\angle EUL = 90^\circ$ creating a $30-60-90$ right triangle.

b. $m\angle UEL = 30^\circ$ because the $\angle E$ was bisected.

c. $m\angle ELQ = 60^\circ$ because the original triangle is equilateral, thus it is equiangular. Algebraically, $3x = 180^\circ$, so $x = 60^\circ$.

5. Statement a must be true because angles opposite congruent sides are congruent. Statement $c$ must be true because two angles in a triangle cannot equal 180° because all three must have that sum.

6. Statement $b$ is possible, but not always true. An isosceles triangle could have 2 30° base angles, which would not be complementary. Statement $d$ must always be false because all three angles in a triangle must add up to 180°.

7. $a = 180 - 134 = 46^\circ$ (Linear Pair Postulate)

8. $b = 88^\circ$ The triangle that includes $a$ and $b$ is isosceles. Therefore, $b = 180 - (2 * 46) = 88^\circ$

9. $c = 46^\circ$ because the measure of the other base angle in the isosceles triangle is $46^\circ$, so by the Corresponding Angles Postulate, $c = 46^\circ$.

10. $d = 134^\circ$ because of the Linear Pair Postulate

11. $e = 46°$ (either by Linear Pair Postulate or Vertical Angles Theorem)

12. $f = 67°$ because $d$ and the vertical angle where $f$ is are congruent. That angle is bisected, creating $f$ and the angle with an equal measure, both 67°.

13. $g = 67°$ (Corresponding Angles Postulate)

## Congruence Transformations

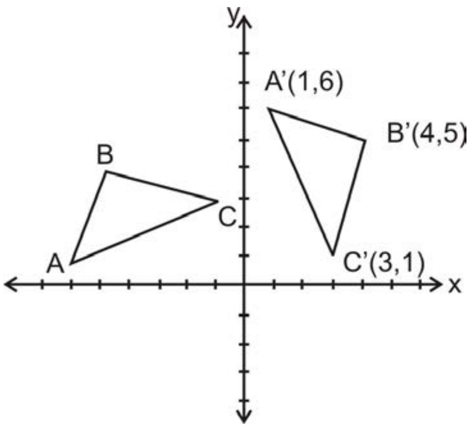
1. Since $A(-6,1), B(-5,4)$, and $C(-1,3)$ and we want to move the triangle down 3 units, we can subtract 3 from the $y-$coordinates of all the existing points.

$A' = (-6, 1-3),\ B' = (-5, 4-3),\ C' = (-1, 3-3) \rightarrow A'(-6,-2),\ B'(-5,1)$, and $C'(-1,0)$.

2. We want to slide up 2 units, so we add 2 to the $y-$coordinates, and we want to move 5 units to the right, so we add 5 to the $x-$coordinates. $A' = (-6+5, 1+2),\ B' = (-5+5, 4+2),\ C' = (-1+5, 3+2) \rightarrow A'(-1,3),\ B'(0,6)$, and $C'(4,5)$.

3. Since we want to reflect across the $y-$axis, the $y-$coordinates will not change, and the $x-$coordinates will change signs. $A'(6,1),\ B'(5,4)$, and $C'(1,3)$.

4. $A(-6,1),\ B(-5,4)$, and $C(-1,3)$



5. This is a reflection about the $x-$axis.

6. $PQ = \sqrt{(2-0)^2 + (-3-3)^2} = \sqrt{2^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40}$
$P'Q' = \sqrt{(-2-0)^2 + (-3-3)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40}$

7. Yes, it is congruence conserving. We know this because all corresponding sides are congruent.

8. This is a dilation.

9. No, a dilation creates ratios of the sides. In this case, the sides of the larger triangle are twice as long as the sides of the smaller triangle.

10. Yes, a rotation of 180° is the same as doing two consecutive rotations about the origin. The rotations must be one across the $x-$axis and then one across the $y-$axis.

If the coordinates of a point are $(x, y)$, then the coordinates of the point after a 180° rotation are $(-x, -y)$.

Now, suppose we reflect the point $(x, y)$ two times, first across the $x-$axis, and then the $y-$axis. When the point is reflected across the $x-$axis, the new coordinates would be $(x, -y)$. Then, if we reflect that point across the $y-$axis, the new coordinates would be $(-x, -y)$.

These final coordinates are the same as the coordinates of a point rotated 180° about the origin.

# Chapter 5

# TE Relationships Within Triangles - Solution Key

## 5.1 Relationships with Triangles

### Midsegments of a Triangle

1. $RS = \frac{1}{2}OP = \frac{1}{2}(12) = 6; TU = \frac{1}{2}OP = \frac{1}{2}(12) = 6$

2. If $RS = 8$, then $TU = 8$.

3. $RS = \frac{1}{2}OP$, so by substitution, $2x = \frac{1}{2}(18) \Rightarrow 2x = 9 \Rightarrow x = 4.5$. $TU = 9$.

4. $RS = \frac{1}{2}OP$ so by substitution, $6x - 8 = \frac{1}{2}(4x)$.

$$\begin{aligned} 6x - 8 &= 2x \\ 4x - 8 &= 0 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

5. a. $M_{\overline{XZ}} = \left(\frac{1+3}{2}, \frac{1+9}{2}\right) = (2, 5)$

b. According to the Midsegment Theorem, the midsegment joins the midpoints of two sides of a triangle. So, we can find the midpoint of $\overline{YZ}$ which will be point $N$.

$$M_{\overline{YZ}} = \left(\frac{5+3}{2}, \frac{5+9}{2}\right) = N(4, 7)$$

6. One could calculate the midpoint of one of the sizes $\overline{YX}$ or $\overline{YZ}$, then use the Parallel Postulate to find the equation of the line that is parallel to $\overline{XZ}$ and passes through the midpoint. It will also go through the other midpoint.

7. By the definition of the midpoint, we know the two segments on the right side of the triangle are congruent, which means they have equal measures. So,

$$\begin{aligned}4y &= 6(y-1)\\4y &= 6y-6\\0 &= 2y-6\\6 &= 2y\\3 &= y\end{aligned}$$

To find $x$, we need to remember the relationship between the midsegment and the third side. The Midsegment Theorem states the midsegment is half as long as the third side, so

$$\begin{aligned}x+4 &= \frac{1}{2}(4x-2)\\x+4 &= 2x-1\\4 &= x-1\\5 &= x\end{aligned}$$

8. By the definition of the midpoint, we know the two segments on the top side of the triangle are congruent, which means they have equal measures. So,

(1)

$$2x-2y=7$$

We also need to remember the relationship between the midsegment and the third side. The Midsegment Theorem states the midsegment is half as long as the third side, so

(2)

$$7=\frac{1}{2}(x+2y)$$

We must solve the system of equations with equations (1) and (2).

Solving for $x$ in equation (2) gives $7=\frac{1}{2}x+y \Rightarrow x=2(7-y) \Rightarrow x=14-2y$.

Then substituting this value for $x$ into equation (1): $2(14-2y)-2y=7$.

Simplifying yields $28-4y-2y=7 \Rightarrow 21=6y \Rightarrow y=\frac{21}{6}=\frac{7}{2}$.

Now, we can substitute this value for $y$ into either of the original equations (1) or (2):

$$2x-2\left(\frac{7}{2}\right)=7 \Rightarrow 2x-7=7 \Rightarrow 2x=14 \Rightarrow x=7$$

9.Since $\triangle RST$ is formed by joining the midpoints of $\triangle XYZ$ , then the length of each side of $\triangle RST$ will be half of the length of the sides of $\triangle XYZ$. This means the perimeter of $\triangle RST$ will be half the perimeter of $\triangle XYZ$. In equation form,

$$P_{\triangle RST}=\frac{1}{2}P_{\triangle XYZ}=\frac{1}{2}(26+38+42)=53\text{ units}$$

10. a. Most of the legwork was completed in #9. $P_{\triangle XYZ} = 26 + 38 + 42 = 106$ units. The relationship, as stated in item 9, is $P_{\triangle RST} = \frac{1}{2} P_{\triangle XYZ}$.

b. The perimeter of a triangle formed by connecting the midpoints of another triangle is half the perimeter of the original triangle.

11.

Table 5.1

| Statement | Reason |
|---|---|
| 1. $A$ is the midpoint of $\overline{OX}$ | 1. Given |
| 2. $\overline{XA} \cong \overline{OA}$ | 2. definition of midpoint |
| 3. $\overline{AB} \times \overline{XY}$ | 3. Given |
| 4. $\overline{OB} \cong \overline{YB}$ | 4. If a parallel line cuts congruent segments in one transversal, then it cuts congruent segments in all transversals. |
| 5. $B$ is the midpoint of $\overline{OY}$ | 5. definition of a midpoint |
| 6. $\overline{BC} \times \overline{YZ}$ | 6. Given |
| 7. $\overline{OC} \cong \overline{ZC}$ | 7. If a parallel line cuts congruent segments in one transversal, then it cuts congruent segments in all transversals. |
| 8. $C$ is the midpoint of $\overline{OC}$ | 8. definition of a midpoint |
| 9. $\overline{AC}$ is a midsegment of $\triangle XYZ$ | 9. definition of a midsegment |
| 10. $\overline{AC} \times \overline{XZ}$ | 10. Midsegment Theorem (5-1) |

12.

Table 5.2

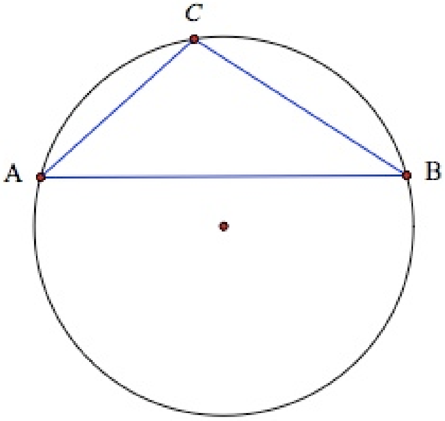
| Statement | Reason |
|---|---|
| 1. $A$ is the midpoint of $\overline{OX}$ | 1. Given |
| 2. $\overline{XA} \cong \overline{OA}$ | 2. definition of midpoint |
| 3. $\overline{AC} \times \overline{XZ}$ | 3. Given |
| 4. $\overline{OC} \cong \overline{ZC}$ | 4. If a parallel line cuts congruent segments in one transversal, then it cuts congruent segments in all transversals. |
| 5. $C$ is the midpoint of $\overline{OC}$ | 5. definition of a midpoint |
| 6. $\overline{AC}$ is a midsegment of $\triangle XYZ$ | 6. definition of a midsegment |
| 7. $XZ = 2AC$ | 7. Midsegment Theorem (5-1) |
| 8. $\overline{AB} \times \overline{XY}$ | 8. Given |
| 9. $\overline{OB} \cong \overline{YB}$ | 9. If a parallel line cuts congruent segments in one transversal, then it cuts congruent segments in all transversals. |
| 10. $B$ is the midpoint of $\overline{OY}$ | 10. definition of a midpoint |
| 11. $\overline{AB}$ is a midsegment of $\triangle XYZ$ | 11. definition of a midsegment |
| 12. $XY = 2AB$ | 12. Midsegment Theorem (5-1) |
| 13. $\triangle OAC \cong \triangle OBA$ | 13. given |
| 14. $\overline{AC} \cong \overline{AB}$ | 14. CPCTC |
| 15. $AC = AB$ | 15. definition of congruent segments |
| 16. $XZ = XY$ | 16. Substitution |

Table 5.2: (continued)

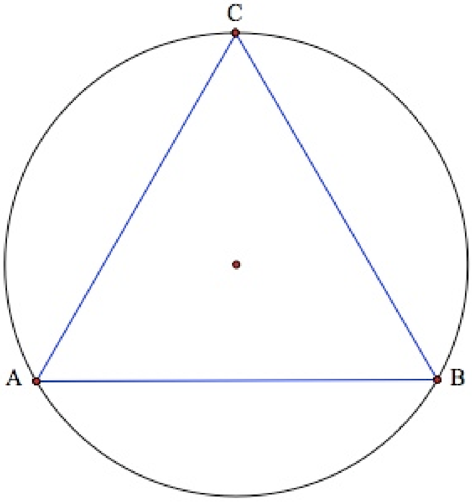
| Statement | Reason |
|---|---|
| 17. $\overline{XZ} \cong \overline{XY}$ | 17. definition of congruent segments |
| 18. $\overline{XO} \cong \overline{XO}$ | 18. Reflexive property of segment congruence |
| 19. $\triangle OXZ \cong \triangle OYX$ | 19. SSS Triangle Congruence Postulate |

## Perpendicular Bisectors in Triangles

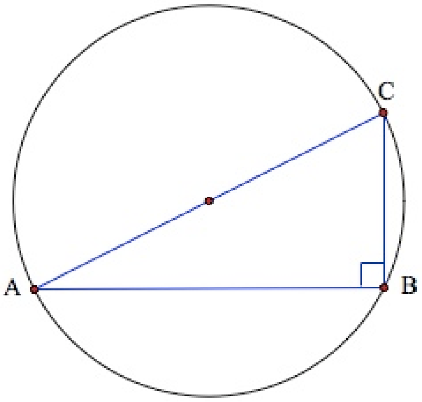
1.



2.

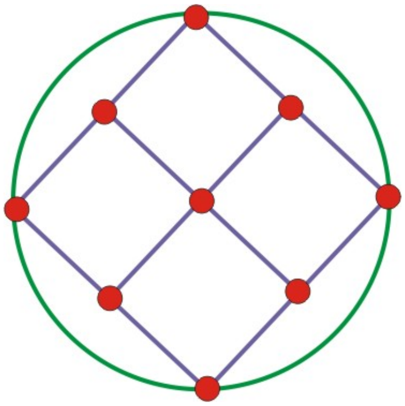


3.

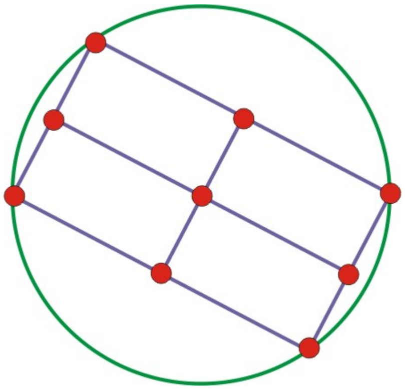


4. The circumcenter of a triangle may lie on the triangle, inside the triangle, or outside the triangle. The circumcenter of an acute triangle lies inside the triangle; the circumcenter of a right triangle lies on the hypotenuse of the triangle; the circumcenter of an obtuse triangle lies outside the triangle.

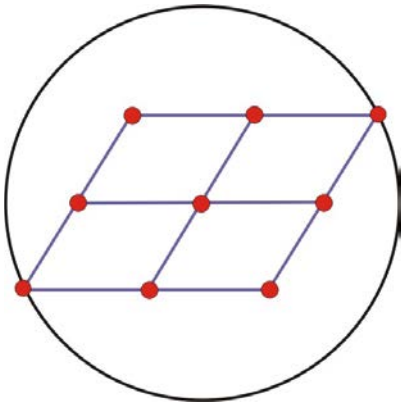
5. a. a square - Yes



b. a rectangle - Yes

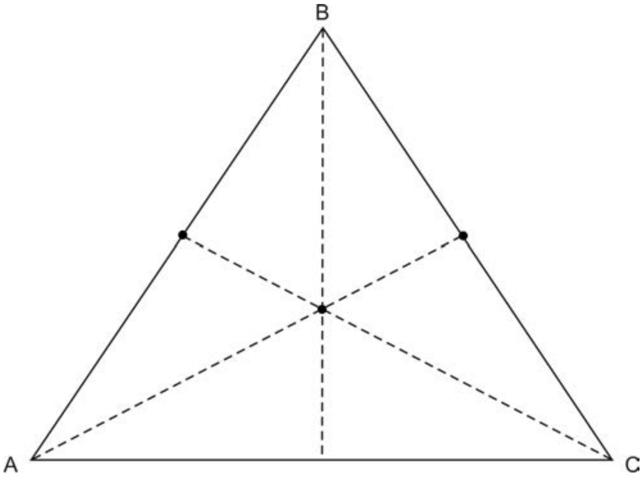


c. a parallelogram - No



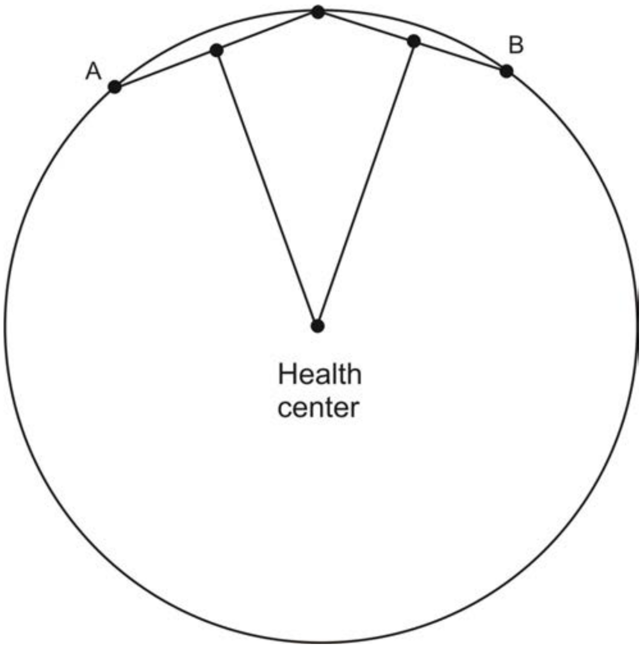
d. For a quadrilateral to be circumscribed, the opposite angles must be supplementary.

6.



We can conclude the six triangles are congruent. We can also conclude they are $30^\circ - 60^\circ - 90^\circ$ right triangles.

7. This plan is not wise. Since the triangle formed by the three cities creates an obtuse triangle, the circumcenter, where the health center would need to be to be equidistant, would lie outside the triangle. The circumcenter would be much further from the cities than the distance between the cities themselves. See the figure below for clarification.



8. This statement is false. Obtuse and right triangles can be isosceles too, and the circumcenters for them are not inside the triangle. Refer to the answer to #4 for more information.

9. This statement is true. Refer to #10 for explanation.

10. First, since we know the coordinates of $A$ and $B$, we can calculate the slope. $m_{\overline{AB}} = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$. Any perpendicular line to side $\overline{AB}$ must have a slope of $-2$, so $m_{\perp} = -2$. Now, since the line we are looking for is a bisector, we should calculate the midpoint of side $\overline{AB}$ . $M_{\overline{AB}} = \left(\frac{2+6}{2}, \frac{1+3}{2}\right) = (4, 2)$.

Now we know a point on the line and the slope of the line, so we can determine the equation of the perpendicular bisector using $y - y_1 = m(x - x_1)$. $y - 2 = -2(x - 4) \rightarrow y - 2 = -2x + 8 \rightarrow y = -2x + 10$. We know the sides of the triangle are have the same length since it is an equilateral triangle. If we calculate the distance from $A$ to $B$, we will have the distance from $A$ to $C$, or $AC$.

$$AC = AB = \sqrt{(2-6)^2 + (1-3)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}.$$

Since $C$ lies on $y = -2x + 10$, the coordinates can be represented by the coordinates $C(x, -2x + 10)$. The value of $x(2\sqrt{5})$ can be rounded to $x = 4.47$. Substituting that value in to the equation gives $y = 1.06$). Therefore, the coordinates of $C$ are approximately $(4.47, 1.06)$

11.

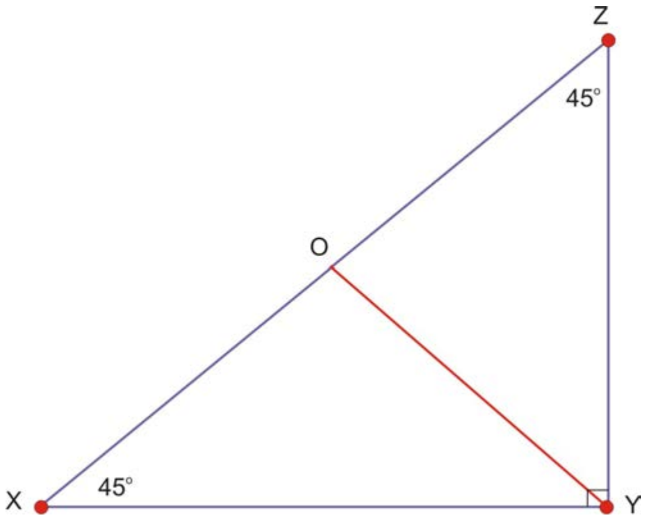
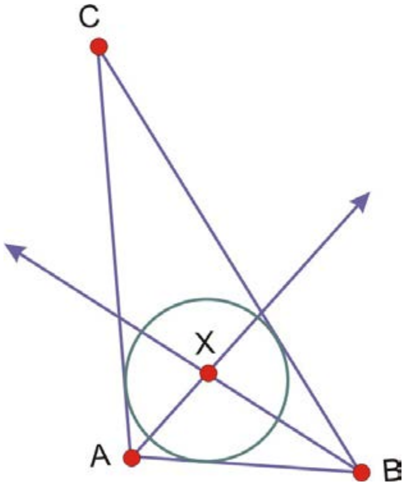


Table 5.3

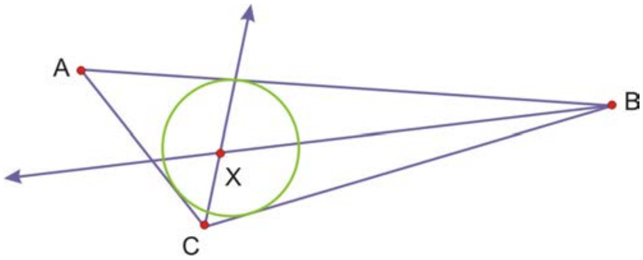
| Statement | Reason |
|---|---|
| 1. $\triangle XYZ$ is a $45-45-90$ right triangle | 1. Given |
| 2. $\angle X \cong \angle Z$ | 2. Given |
| 3. $\overline{XY} \cong \overline{ZY}$ | 3. angles opposite congruent angles are congruent |
| 4. $\overline{OY}$ is the angle bisector of $\angle XYZ$ | 4. by construction |
| 5. $\angle XYO \cong \angle ZYO$ | 5. definition of angle bisector |
| 6. $\triangle XYO \cong \triangle ZYO$ | 6. ASA Triangle Congruence Postulate |
| 7. $\overline{XO} \cong \overline{ZO}$ | 7. CPCTC |
| 8. $O$ is the midpoint of $\overline{XZ}$ | 8. definition of midpoint |
| 9. $\overline{OY}$ is the perpendicular bisector of $\triangle XYZ$ | 9. definition of perpendicular bisector |

## Angle Bisectors in Triangles

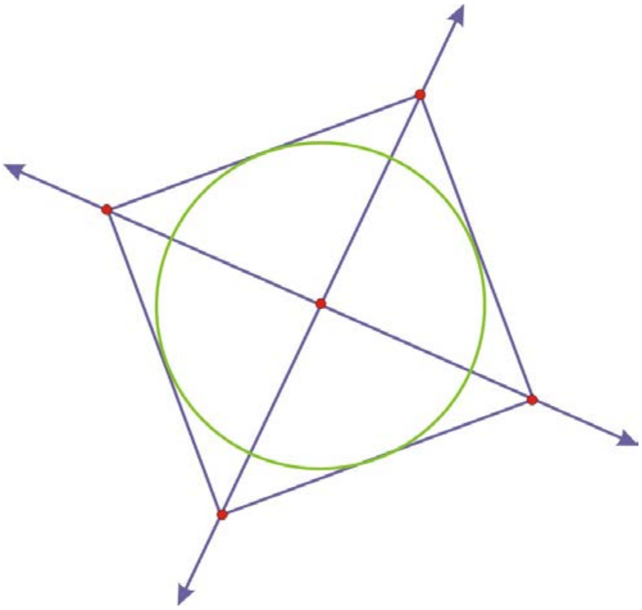
1.



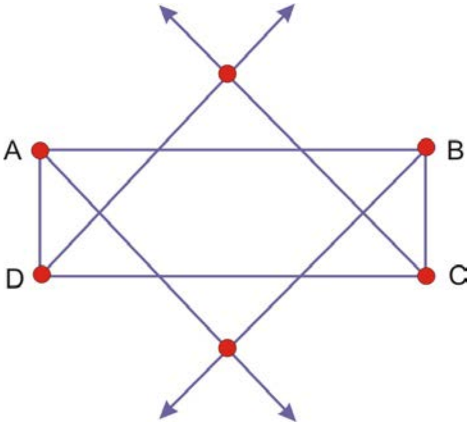
2.



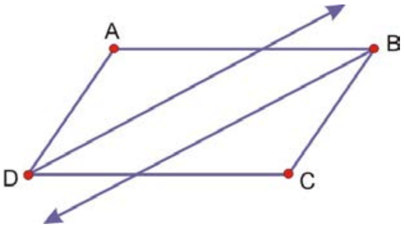
3. a. a square



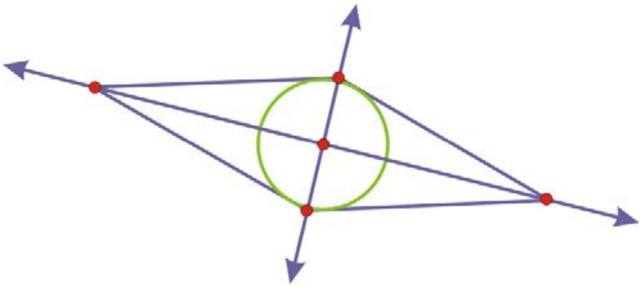
b. a rectangle



c. a parallelogram

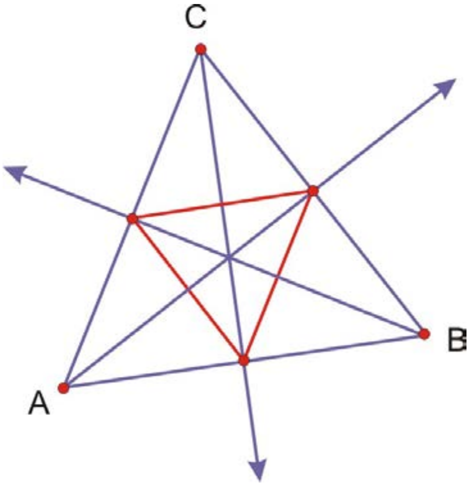


d. a rhombus

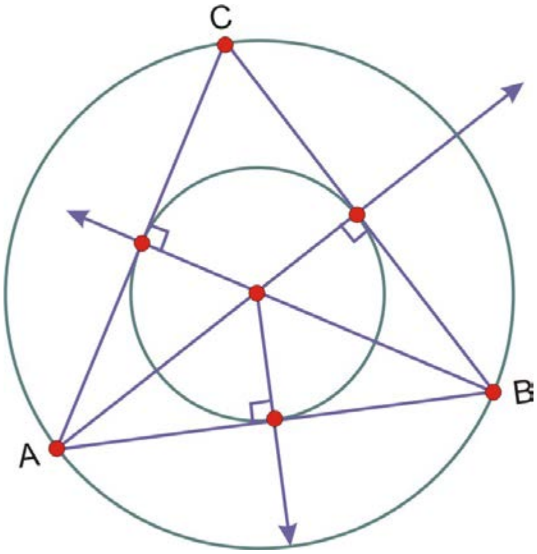


e. In order to circumscribe a quadrilateral, the angle bisectors must be concurrent.

4.



5. This statement is true; the incenter and circumcenter of an equilateral triangle are the same as are a square's incenter and circumcenter.



6. One can conclude there are 4 congruent triangles and a separate pair of congruent triangles inside the isosceles triangle.

7. Remember from #3 that to circumscribe a quadrilateral, the angle bisectors must be congruent. The figure for 3a shows a square circumscribed, and from #5, we can see it can also be inscribed, so let's work with it.

The distance from $A$ to $B, AB = |8-4| = 4$ units. Since $\overline{AB}$ is a horizontal line, the slope is 0. Therefore, $\overline{AC}$ would be a vertical line segment ending 4 units up from $A$, so $C(4, 4+4) = (4, 8)$. $\overline{BD}$ is also a vertical line segment ending 4 units up from $B$, so $D(8, 4+4) = (8, 8)$.

One can use a similar process to find the coordinates for a kite, which would be $C(6, 6)$ and $D(6, 2)$. The kite can be inscribed, but not circumscribed.

8. The fold line is a line of symmetry for the triangle because it divides the triangle into two congruent triangles. If you repeated the process for the other two sides, you would get the same result.

9. Only the line from the vertex angle creates congruent triangles, thus is the only line of symmetry.

10. You can create congruent figures inside any regular polygon using the method from #8 and #9.

11.

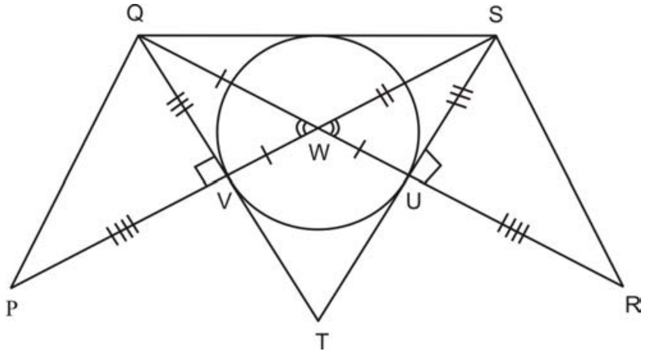


Table 5.4

| Statement | Reason |
|---|---|
| 1. $\overline{ST}$ is the perpendicular bisector of $\overline{QR}$, $\overline{QT}$ is the perpendicular bisector of $\overline{SP}$ | 1. Given |
| 2. $\angle SUW$ and $\angle QVW$ are right angles. | 2. definition of perpendicular bisector |
| 3. $\angle SUW \cong \angle QVW$ | 3. All right angles are congruent. |
| 4. $W$ is the incenter and circumcenter of $\triangle QST$ | 4. definition of incenter and circumcenter |
| 5. $\overline{QW} \cong \overline{SW}$ | 5. Concurrency of Perpendicular Bisectors Theorem |
| 6. $QW = SW$ | 6. definition of congruent segments |
| 7. $\overline{VW}$ and $\overline{UW}$ are radii of circle $W$. | 7. definition of radii |
| 8. $\overline{VW} \cong \overline{UW}$ | 8. Radii of the same circle are congruent. |
| 9. $\triangle QVW \cong \triangle SUW$ | 9. HL Triangle Congruence Theorem |
| 10. $\overline{QV} \cong \overline{SU}$ | 10. CPCTC |
| 11. $QW + WU = QU, SW + WV = SV$ | 11. Segment Addition Postulate |
| 12. $SW + WU = QU$ | 12. Substitution |
| 13. $QU = SV$ | 13. Substitution |
| 14. $\overline{QU} \cong \overline{SV}$ | 14. definition of congruent segments |
| 15. $\overline{QU} \cong \overline{RU}, \overline{SV} \cong \overline{PV}$ | 15. definition of perpendicular bisector |
| 16. $\overline{PV} \cong \overline{RU}$ | 16. Transitive Property of Segment Congruence |
| 17. $\triangle QVR \cong \triangle SUR$ | 17. SAS Triangle Congruence Postulate |
| 18. $\overline{PQ} \cong \overline{RS}$ | 18. CPCTC |

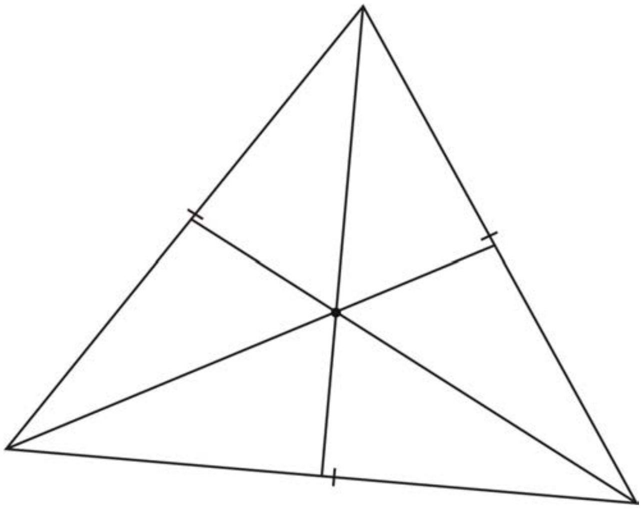
12.

Table 5.5

| Statement | Reason |
|---|---|
| 1. $\overline{PQ}$ bisects $\angle XQR$ | 1. Given |
| 2. $\angle XQP \cong \angle PQR$ | 2. definition of angle bisector |
| 3. $\overline{PR}$ bisects $\angle QRZ$ | 3. Given |
| 4. $\angle ZRP \cong \angle PRQ$ | 4. definition of angle bisector |
| 5. Draw a line segment from $X$ to $Z$. | 5. Line postulate |
| 6. Construct perpendicular segments $\overline{PA}, \overline{PB}, \overline{PC}$. | 6. Line postulate |
| 7. $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ | 7. Angle Bisector Theorem |
| 8. $\overline{PY}$ bisects $\angle XYZ$ | 8. Converse of Angle Bisector Theorem |

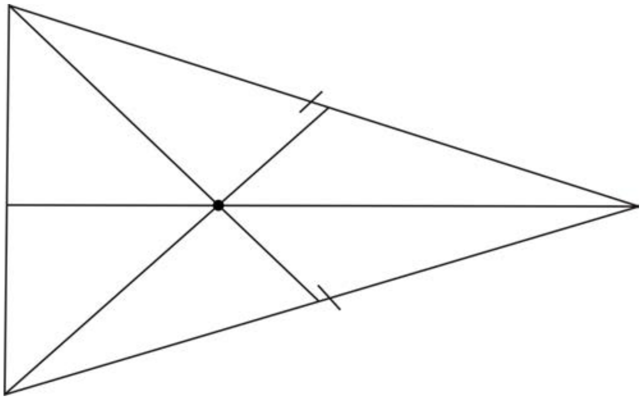
## Medians in Triangles

1. a. an equilateral triangle



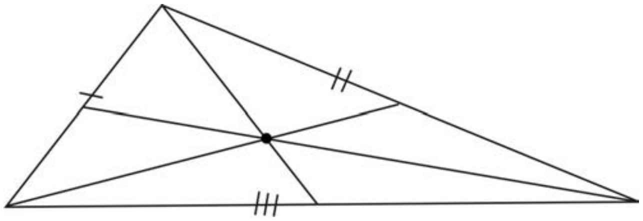
The medians all have the same length. The distance from each vertex to the centroid is the same, which is two-thirds the length of the median.

b. an isosceles triangle



Two of the medians have the same length. The distance from each vertex to the centroid is the same for these. The distance from each vertex to the centroid is two-thirds the length of the median.

c. a scalene triangle



All the median are different lengths, as is the distance from each vertex to the centroid. The distance from each vertex to the centroid is two-thirds the length of the median.

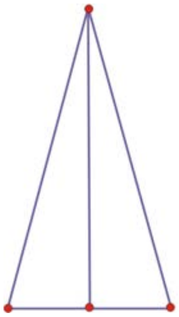
2. Since $X$ is the centroid of $\triangle ABC$, then the following relationships hold true:

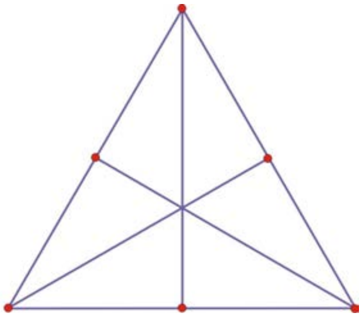
$XC = \frac{2}{3}CS$ and $XS = \frac{1}{3}CS$

From these equations, we can see (or calculate) $XC = 2XS$. Since $XS = 10, XC = 2(10) = 20$.

The Segment Addition Postulate states $XC + XS = CS \rightarrow 10 + 20 = CS = 30$.

3. This statement is false because a median can be the angle bisector of the vertex angle if an isosceles triangle and all three angles of an equilateral triangle.





4. We can find the mean of the $x-$coordinates of the vertices of triangle $\triangle ABC$ to find the $x-$coordinate of the centroid, and find the mean of the $y-$coordinates of the vertices of triangle $\triangle ABC$ to find the $y-$coordinate of the centroid.

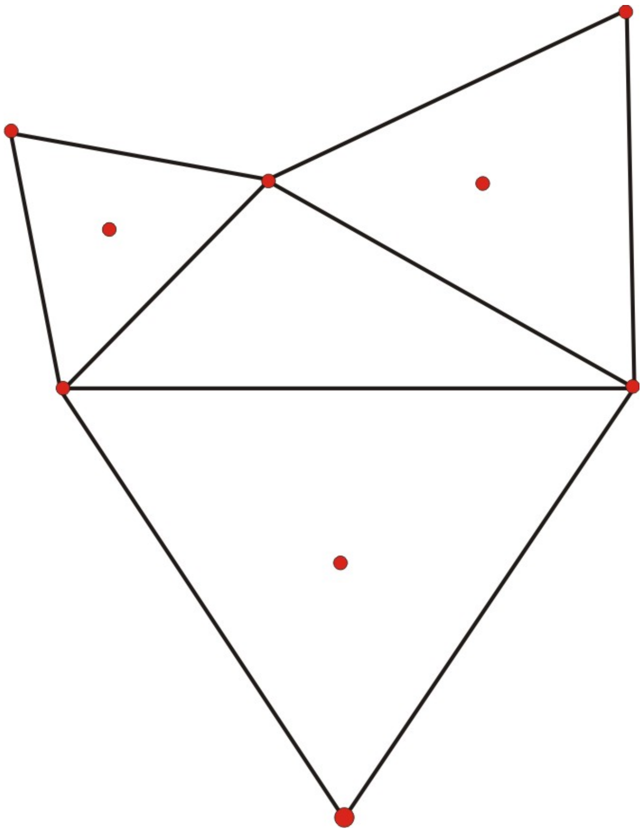
Then, the centroid is $\left(\frac{2+4+8}{3}, \frac{3+1+5}{3}\right) = X\left(\frac{14}{3}, 3\right)$

5. a. We can follow the same steps as in #4 to find the centroid of triangle $\triangle ABC$.

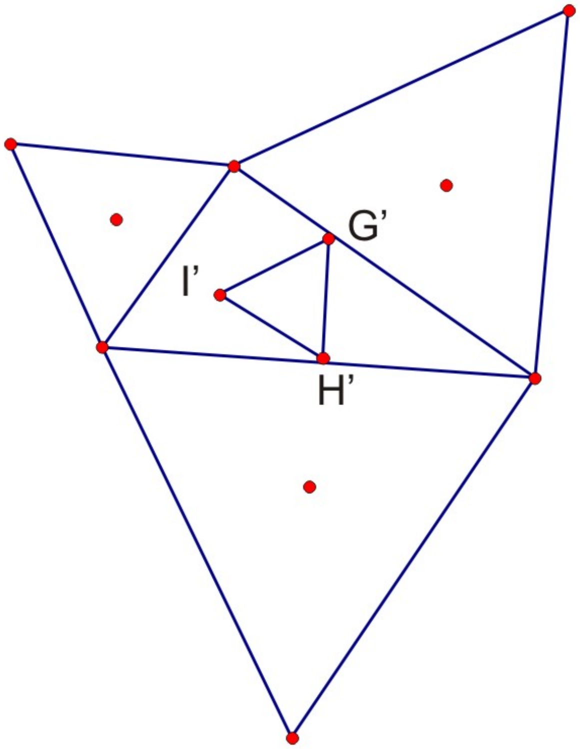
Then, centroid is $\left(\frac{1+5+6}{3}, \frac{1+2+6}{3}\right) = X(4, 3)$

b. $XB = \sqrt{(5-4)^2 + (2-3)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

6. a.



b.

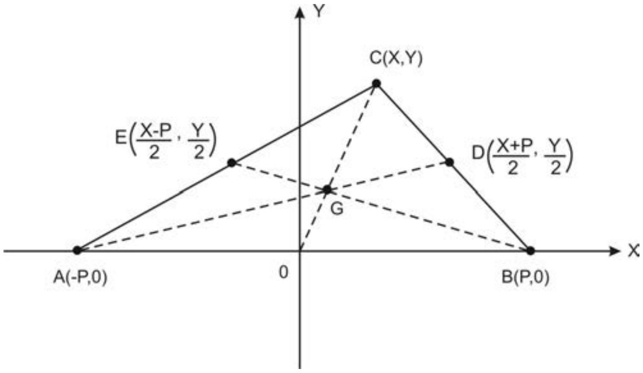


c. The triangle is equilateral.

7. The centroid can be found the same way as in #4.

$$\text{centroid} = \left(\frac{1+1+10}{3}, \frac{0+8+4}{3}\right) = (4,4)$$

8.


Y
C(X,Y)
E$\left(\frac{X-P}{2}, \frac{Y}{2}\right)$
D$\left(\frac{X+P}{2}, \frac{Y}{2}\right)$
G
X
A(-P,0)
0
B(P,0)

We are given the coordinates $A(-p,0), B(p,0)$, and $C(x,y)$. Then, using the method from 7, we can calculate the coordinates of the centroid $G$ as $\left(\frac{-p+x+p}{3}, \frac{0+y+0}{3}\right)$ which simplifies to $G\left(\frac{1}{3}x, \frac{1}{3}y\right)$. $G$ is $\frac{2}{3}$ the distance from $C$ to $O$.

The coordinates of $D$, which we define as the midpoint of $\overline{BC}$ are $\left(\frac{x+p}{2}, \frac{y}{2}\right)$. Then the distance from $A$ to $D$,

$AD = \sqrt{\left(\frac{x+p}{2} - (-p)\right)^2 + \left(\frac{y}{2} - 0\right)^2} = \sqrt{\left(\frac{x+3p}{2}\right)^2 + \left(\frac{y}{2}\right)^2} = \frac{\sqrt{x^2+6px+9p^2+y^2}}{2}$. $AG = \sqrt{\left(\frac{x}{3} - (-p)\right)^2 + \left(\frac{y}{3} - 0\right)^2} = \sqrt{\left(\frac{x+3p}{3}\right)^2 + \left(\frac{y}{3}\right)^2} = \frac{\sqrt{x^2+6px+9p^2+y^2}}{3}$ , which is $\frac{2}{3}AD$.

The coordinates of $E$, which we define as the midpoint of $\overline{AC}$ are $\left(\frac{x-p}{2}, \frac{y}{2}\right)$. $BE = \sqrt{\left(\frac{x-p}{2} - p\right)^2 + \left(\frac{y}{2} - 0\right)^2} = \sqrt{\left(\frac{x-3p}{2}\right)^2 + \left(\frac{y}{2}\right)^2} = \frac{\sqrt{x^2-6px+9p^2+y^2}}{2}$. $BG = \sqrt{\left(\frac{x}{3} - p\right)^2 + \left(\frac{y}{3} - 0\right)^2} = \sqrt{\left(\frac{x-3p}{3}\right)^2 + \left(\frac{y}{3}\right)^2} = \frac{\sqrt{x^2-6px+9p^2+y^2}}{2}$, which is $\frac{2}{3}BE$.

9.

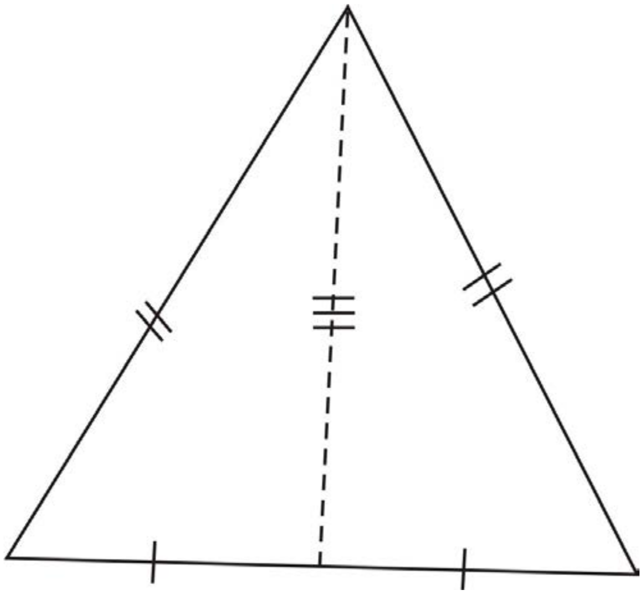
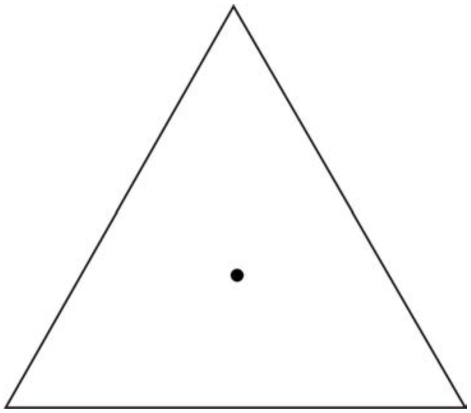


Table 5.6

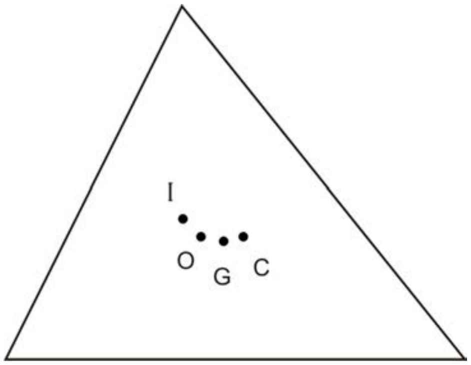
| Statement | Reason |
|---|---|
| 1. Draw equilateral triangle $ABC$ | 1. Given |
| 2. Draw median from $A$ to $\overline{BC}$ | 2. Line Postulate |
| 3. $D$ is the midpoint of $\overline{BC}$ | 3. definition of angle bisector |
| 3. $\overline{AD} \cong \overline{AD}$ | 3. Reflexive Property of Segment Congruence |
| 4. $\overline{BD} \cong \overline{CD}$ | 4. definition of midpoint |
| 5. $\angle BDA \cong \angle CDA$ | 5. Since $\triangle ABC$ is equilateral, $\overline{AD}$ is a perpendicular bisector. |
| 6. $\triangle BDA \cong \triangle CDA$ | 6. ASA Triangle Congruence Postulate |

## Altitudes in Triangles

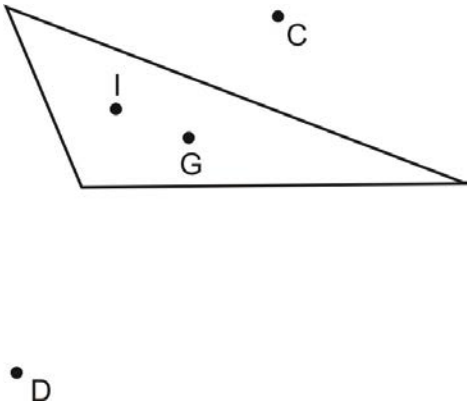
1. From the figure, you can see the 4 points are the same.



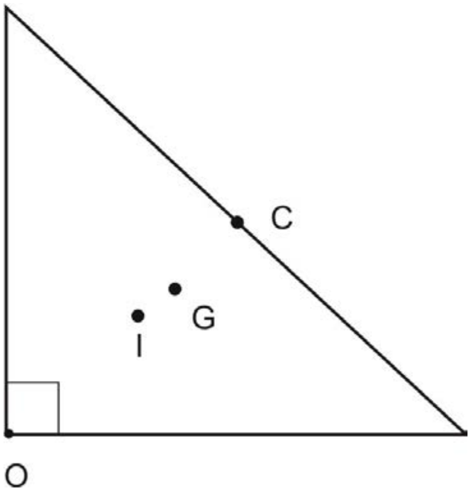
2. From the figure, you can see the 4 points lie inside the triangle.



3. From the figure, you can see two of the points are outside the triangle, and two are in. Typically, two are inside, one is on the triangle, and one (The centroid must always be inside the triangle) is always inside the triangle.



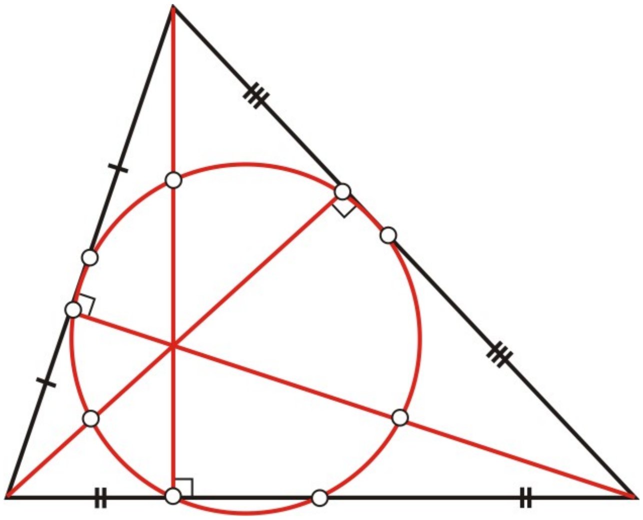
4. From the figure, you can see the orthocenter lies on the vertex of the right angle and the circumcenter lies on the midpoint of the hypotenuse.



5. The points for the orthocenter, circumcenter, and the centroid always lie on the same line. (They are collinear.) Also, the centroid is always inside the triangle.

6. One, you can determine that the circumcenter and the orthocenter are the endpoints of the Euler segment. Also, you can conclude that the distance from the centroid to the circumcenter is half the distance from the orthocenter to the centroid.

7.



`http://en.wikipedia.org/wiki/File:Triangle.NinePointCircle.svg`

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Three of the points are the where the altitudes intersect the opposite sides of the triangle. Three other points are the midpoints of the sides of the triangle, and the last three points are the midpoints of the segments connecting the orthocenter with the vertex.

8.

Table 5.7

| Statement | Reason |
|---|---|
| 1. $\triangle ABC \cong \triangle DEF$ | 1. Given |
| 2. $\overline{AP}$ is the altitude of $\triangle ABC$ $\overline{DO}$ is the altitude of $\triangle DEF$ | 2. Given |
| 3. $\overline{AB} \cong \overline{DE}$ | 3. CPCTC |
| 4. $\angle BAC \cong \angle EDF, \angle C \cong \angle F$ | 4. CPCTC |
| 5. $m\angle ABP = m\angle BAC + m\angle C$ <br> $m\angle DEO = m\angle EDF + m\angle F$ | 5. Remote Angles of a Triangle Theorem |
| 6. $m\angle DEO = m\angle BAC + m\angle C$ | 6. Substitution |
| 7. $\angle ABP \cong \angle DEO$ | 7. definition of congruent angles |
| 8. $\triangle APB \cong \triangle DOE$ | 8. AAS Triangle Congruence Postulate |
| 9. $\overline{AP} \cong \overline{DO}$ | 9. CPCTC |

9.

Table 5.8

| Statement | Reason |
|---|---|
| 1. $\triangle ABC$ is isosceles | 1. Given |
| 2. $\overline{AB} \cong \overline{AC}, \overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ | 2. Given |
| 3. $\angle ABC \cong \angle ACB$ | 3. Base Angles Theorem |
| 4. $\overline{BC} \cong \overline{CB}$ | 4. Symmetric Property of Congruence |
| 5. $\triangle BEC \cong \triangle CDB$ | 5. AAS Triangle Congruence Postulate |
| 6. $\overline{BD} \cong \overline{CE}$ | 6. CPCTC |

## Inequalities in Triangles

1. a. The largest angle is $\angle R$ because it is opposite the longest side. The smallest angle is $\angle S$ because it is opposite the shortest side.

b. The largest angle is $\angle C$ because it is opposite the longest side. The smallest angle is $\angle A$ because it is opposite the shortest side.

2. a. $m\angle B = 180 - (60 + 55) = 65°$, so the longest side is opposite $\angle B$ , which is $\overline{AC}$ . The shortest side is opposite the smallest angle, so $\overline{AB}$.

b. $m\angle B = 180 - (90 + 43) = 47°$ The longest side is opposite $\angle A$ , which is $\overline{BC}$ . The shortest side is opposite the smallest angle, so $\overline{AB}$ .

3. a. According to the Triangle Inequality Theorem, any side of a triangle cannot exceed the sum of the two remaining sides. All three combinations of sides gives

$$6 + 6 = 12$$
$$13 + 6 = 19$$

Since $13 > 12$, it is not possible to form a triangle with these measurements. b.

$$\begin{aligned} 8+9&=17\\ 10+9&=19\\ 10+8&=18 \end{aligned}$$

Since all sides fall within the acceptable ranges, it is possible to form a triangle.

c.

$$\begin{aligned} 18+7&=25\\ 11+7&=18 \end{aligned}$$

It is not possible to create this triangle because $11 + 7 = 18$.

d.

$$\begin{aligned} 3+4&=7\\ 4+5&=9\\ 3+5&=8 \end{aligned}$$

Since all sides fall within the acceptable ranges, it is possible to form a triangle.

4. The third side must fall between $24 - 18$ and $24 + 18$, so $6 < x < 42$.

5. We know the legs have the same length, so we will can that length $x$. Then, $2x > 30$, or $x > 15$. Each leg must be longer than 15.

6. Angle 2 is the largest angles because it is opposite the longest side.

7. Angle 1 must be the largest angle because $y + 1$ would be longer than the other sides.

8. By the Triangle Sum Theorem, $x + (x + 4) + 60 = 180 \rightarrow 2x + 64 = 180 \rightarrow 2x = 116 \rightarrow x = 58^\circ$. So, $m\angle D = 58^\circ, m\angle E = 62^\circ, m\angle F = 60^\circ$. The longest side is $\overline{DF}$.

9. $m\angle W = 60^\circ$ , so all sides have the same measure in $\triangle XWZ$. $m\angle Y = 60^\circ$, so side $\overline{XY}$ is the longest side in $\triangle XYZ$.

10.

Table 5.9

| Statement | Reason |
|---|---|
| 1. $m\angle Y > m\angle X, m\angle S > m\angle R$ | 1. Given |
| 2. $XO > YO, RO > SO$ | 2. Triangle Inequality Theorem |
| 3. $XO + RO = XR, YO + SO = YS$ | 3.Segment Addition Postulate |
| 4.$XR > YS$ | 4. Substitution |

11.

Table 5.10

| Statement | Reason |
|---|---|
| 1. $\overline{XO} \cong \overline{OY} \cong \overline{YZ}$ | 1. Given |
| 2. $m\angle XOY > m\angle YOZ$ | 2. definition of obtuse and acute angles |
| 3. $XY > ZY$ | 3. Triangle Inequality Theorem |

## Inequalities in Two Triangles

1. By the Hinge Theorem, the side that has a measure of $3x + 1$ is longer than the side with a length of $x + 3$. So,

$$\begin{aligned} 3x + 1 &> x + 3 \\ 3x - x &> 3 - 1 \\ 2x &> 2 \\ x &> 1 \end{aligned}$$

2. Since we know $\angle ABC$ is acute and $\angle DEF$ is obtuse, we know $m\angle DEF > m\angle ABC$. Therefore, by the Hinge Theorem, $DF > AC$.

3. By the SSS Inequality-Converse of Hinge Theorem, $m\angle 1 > m\angle 2$.

4. Since we do not know anything about the included angle or the third side of the triangles, we cannot determine anything.

5. By the Hinge Theorem, we know $NO > LM$ since $m\angle LKM > m\angle NPO$.

6. Yes, these statements are true. First, by the Hinge Theorem, since $10.5 > 10$, we know $m\angle 2 > m\angle 1$. Then, by the Base Angles Theorem, $m\angle 3 > m\angle 4$.

7. The assertion that $x = 15$ is false because if $x = 15$, then the triangle to the right would be equilateral. If a triangle is equilateral, it must also be equiangular. All the angles then would be $\frac{180}{3} = 60^\circ$.

8. This statement is true because of the SSS Inequality-Converse of Hinge Theorem. Since the triangles have two sides congruent, we know

$$\begin{aligned} 5x + 2 &> x + 6 \\ 4x &> 4 \\ x &> 1 \end{aligned}$$

9. Since $QS > UT$, it is not possible for $\angle R \cong \angle V$ by SSS Inequality-Converse of Hinge Theorem.

10. We do not have enough information to determine if the assertion is true or false.

## Indirect Proof

1. Since we want to prove $n$ is even, we need to assume the opposite is true, i.e., that $n$ is odd. Even numbers can be found by $2a$, where $a$ is any integer, so $2a + 1$ will give us odd numbers. If $n = 2a + 1$, then $n^2 = 2a^2 + 4a + 1$, which is an odd number. This contradicts our assumption, thus proving $n$ is even.

2. We first assume that $\triangle ABC$ is an equilateral triangle. Since $\triangle ABC$ is equilateral, it is also equiangular, thus $m\angle A = m\angle B = m\angle C$. This contradicts our given information; therefore, $\triangle ABC$ is not an equilateral triangle.

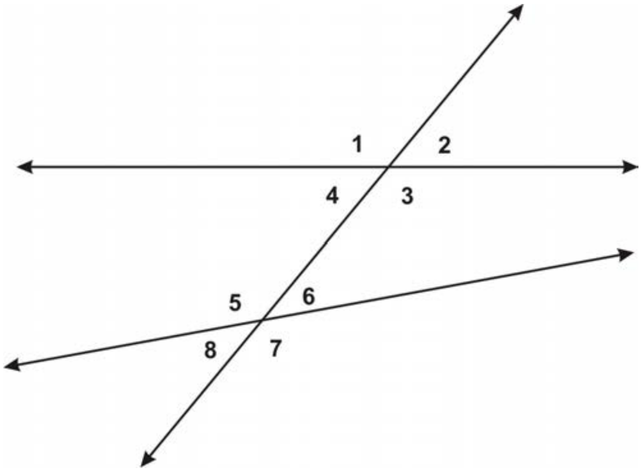
3. Let's assume $x^2 \leq 9$. This inequality gives two scenarios:

(a) $x^2 = 9$, which yields $x = 3$, which is a contradiction.

(b) $x^2 < 9$, which gives us $-3 < x < 3$, which also contradicts our statement.

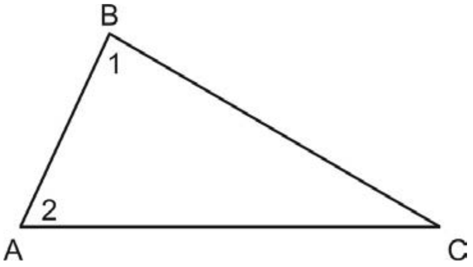
Both statements together contradict our premise, thus proving the original statement true.

4. We will first assume that two lines are not parallel.



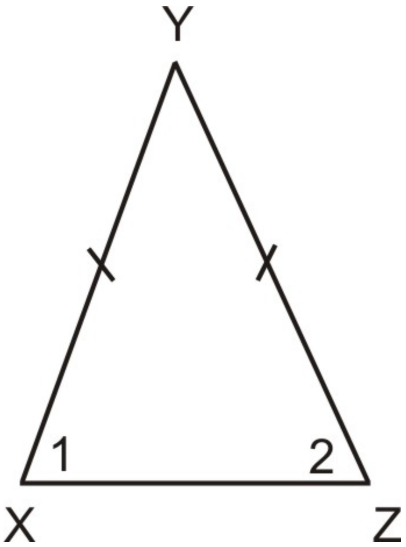
Then, we know $m\angle 4 \neq m\angle 8$. We know $\angle 8 \cong \angle 6$ by the Vertical Angles Theorem, so $m\angle 8 = m\angle 6$ by the definition of congruent angles. This gives us $m\angle 4 \neq m\angle 6$, which contradicts the fact that $\angle 4 \cong \angle 6$. Since we have a contradiction, the original statement must be true.

5. We must first assume a side opposite a larger angle in a triangle is not longer than the side opposite the smaller angle.



$BC = AC$. Then, we know by the Converse of the Triangle Inequality Theorem $m\angle 1 = m\angle 2$. However, this contradicts our given statement that $m\angle 1 > m\angle 2$. So, the original premise must be true.

6. We need to assume that the base angles of an isosceles triangle are not congruent.



So, $\angle 1$ is not congruent to $\angle 2$. By the Triangle Inequality Theorem, $XY \neq ZY$. This contradicts the statement that the triangle is isosceles. Since we have a contradiction, the original statement that the base angles in a triangle are congruent must be true.

7. We will assume $n$ is even. If $n$ is even, it can be represented by $2a$, where $a$ is any integer. Then, $n^2 = 4a^2$, which is an even number. This contradicts the given information that $n^2$ is odd, so $n$ must also be odd.

8. We will first assume that $\angle C$ is a right angle. Then by the definition of a right angle, $m\angle C = 90^\circ$. This gives $m\angle A + m\angle C = 110 + 90 > 180^\circ$. This contradicts the Triangle Sum Theorem, thus we know $\angle C$ cannot be a right angle.

9. We assume that the sides of a triangle are congruent. If two sides of a triangle are congruent, it is an isosceles triangle. By the Isosceles Triangle Theorem, the angles opposite the congruent sides are congruent. This contradicts the statement that the angles are not congruent; therefore, the sides opposite the non-congruent angles are not congruent.

10. We will first assume $\overline{GI}$ bisects $\angle HIJ$. Then, by the definition of an angle bisector, $\angle HIG \cong \angle JIG$. We also know $\overline{GI} \cong \overline{GI}$ by the reflexive property of segment congruence. Thus, we can determine that $\triangle GHI \cong \triangle GJI$ by the SAS Triangle Congruence Postulate. So, by CPCTC, $\overline{HG} \cong \overline{JG}$. This contradicts the given statement, so we know $\overline{GI}$ does not bisect $\angle HIJ$.

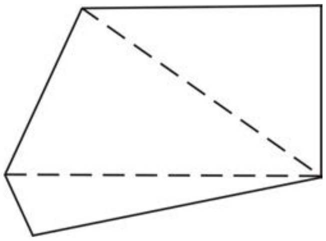
# Chapter 6

# TE Quadrilaterals - Solution Key

## 6.1 Quadrilaterals

### Interior Angles

1.



2. The Triangle Sum Theorem states the sum of the interior angles of a triangle is 180°. There are three triangles in this pentagon, so $3(180^\circ) = 540^\circ$.

3. There are 4 diagonals that can be drawn from one vertex creating 5 interior triangles, so $5(180^\circ) = 900^\circ$

Number of sides $= 7$

Sum of interior angles $= 900^\circ$

4. There are 3 diagonals that can be drawn from one vertex creating 4 interior triangles, so $4(180^\circ) = 720^\circ$

Number of sides $= 6$

Sum of interior angles $= 720^\circ$

5.

Table 6.1

| Polygon name | Number of sides | Sum of measures of interior angles |
|---|---|---|
| Triangle | 3 | 180° |
| quadrilateral | 4 | 360° |
| Pentagon | 5 | 540° |

Table 6.1: (continued)

| Polygon name | Number of sides | Sum of measures of interior angles |
|---|---|---|
| Hexagon | 6 | 720° |
| Heptagon | 7 | 900° |
| Octagon | 8 | 1080° |
| Decagon | 10 | 1440° |
| dodecagon | 12 | 1800° |
| $n-$gon | $n$ | $180(n-2)$ |

6. A pentagon has a sum of interior angles $= 180(5-2) = 540^\circ$. Since there are 5 congruent angles, each angle has a measure of $\frac{540}{5} = 108^\circ$.

7. The sum of the interior angles of any octagon is $1080^\circ$. A regular octagon has each angle measure $\frac{1080}{8} = 135^\circ$

8. Each interior angle in a regular $n-$gon would be the sum of the interior angles divided by the number of angles $= \frac{180(n-2)}{n}$.

9. No, we cannot. In order to use the polygon angle sum theorem, the angles must open into the polygon, not out of it. The interior angle $\angle BCD$ is a reflex angle, thus could not be used in a triangle.

10.

$$\begin{aligned} 180(n-2) &= 2700 \\ n-2 &= 15 \\ n &= 17 \end{aligned}$$

This figure has 17 sides, known as the $17-$gon or a heptadecagon.

## Exterior Angles

1. We can find $y$ first by adding the interior angles up and subtracting their sum from $540^\circ$ since this is a pentagon.

$$y = 540^\circ - (100^\circ + 105^\circ + 57^\circ + 150^\circ) = 128^\circ.$$

Now we can find $x$ using the Linear Pair Postulate.

$$x = 180^\circ - y = 180^\circ - 128^\circ = 52^\circ$$

2. $w = 180^\circ - 110^\circ = 70^\circ$ by the Linear Pair Postulate

$y = 110^\circ$ by the Corresponding Angles Postulate

$x = 70^\circ$ by the Corresponding Angles Postulate

$z = 180^\circ - 90^\circ = 90^\circ$ because of the Same-Side Interior Theorem.

The sum of the interior angles should be 360°. So, $90^\circ + 70^\circ + 90^\circ + 110^\circ = 360^\circ$.

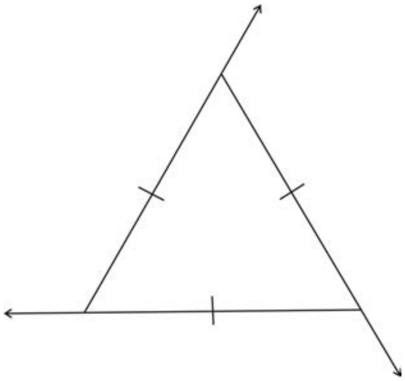
3. We know the measure of the vertex angle is $180^\circ - 145^\circ = 35^\circ$. Then, since the triangle is isosceles, each base angle has a measure of $b$. Now, we can find $b$ by:

$$\begin{aligned} b + b + 35^\circ &= 180^\circ \\ 2b &= 145^\circ \\ b &= 72.5^\circ \end{aligned}$$

We can find a by using the Linear Pair Postulate.

$$a = 180^\circ - b = 180^\circ - 72.5^\circ = 107.5^\circ$$

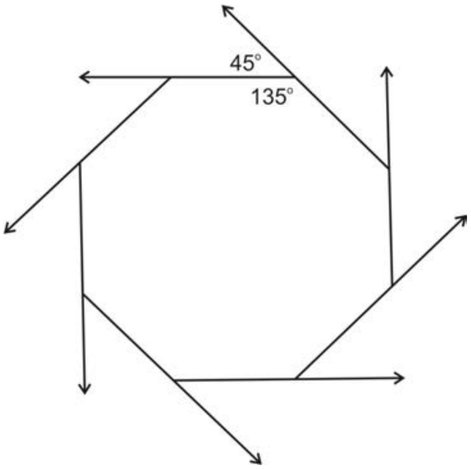
4.



Each exterior angle $= 180^\circ - 60^\circ = 120^\circ$ using the Linear Pair Postulate. The sum of the exterior angles of an equilateral triangle is $120^\circ + 120^\circ + 120^\circ = 360^\circ$.

5. Each interior angle of any polygon can be found by $\frac{180(n-2)}{n}$, where $n$ is the number of sides. So, for an octagon, $\frac{180(n-2)}{n} = \frac{180(8-2)}{8} = 135^\circ$.

6. Each exterior angle will create a linear pair with each interior angle. By the Linear Pair Postulate, the two angles are supplementary. So, each exterior angle is $180^\circ - 135^\circ = 45^\circ$.



7. The sum of the exterior angles of an octagon is $8(45^\circ) = 360^\circ$

8.

Table 6.2

| Polygon name | Number of sides | Sum of measures of interior angles | Measure of each interior angle | Measure of each exterior angle | Sum of measures of exterior angles |
|---|---|---|---|---|---|
| triangle | 3 | 180° | 60° | 120° | 360° |
| quadrilateral | 4 | 360° | 90° | 90° | 360° |
| pentagon | 5 | 540° | 108° | 72° | 360° |
| hexagon | 6 | 720° | 120° | 60° | 360° |
| heptagon | 7 | 900° | 128.57° | 51.43° | 360° |
| octagon | 8 | 1080° | 135° | 45° | 360° |
| decagon | 10 | 1440° | 144° | 36° | 360° |
| dodecagon | 12 | 1800° | 150° | 30° | 360° |
| $n-$gon | $n$ | $180(n-2)^\circ$ | $\frac{180(n-2)^\circ}{n}$ | $\frac{360^\circ}{n}$ | 360° |

9.

$$\begin{aligned}180-\frac{180(n-2)^\circ}{n} &= \frac{360^\circ}{n} \rightarrow\\ 180n-180(n-2) &= 360 \rightarrow\\ 180n-180n+360 &= 360 \rightarrow\\ 360 &= 360\end{aligned}$$

## Classifying Quadrilaterals

1. $y = 180^\circ - 57^\circ = 123^\circ$ because same-side interior angles of parallel lines cut by a transversal are supplementary.

$x = 57^\circ$ because same-side interior angles of parallel lines cut by a transversal are supplementary.

2. $w = 360^\circ - 3(90^\circ) = 90^\circ$ because the sum of the interior angles of a quadrilateral is 360°.

$z = 9$ m because opposite side of rectangles are congruent.

3. $a = 180^\circ - 83^\circ = 97^\circ$ because same-side interior angles of parallel lines cut by a transversal are supplementary.

$b = 180^\circ - 91^\circ = 89^\circ$ because same-side interior angles of parallel lines cut by a transversal are supplementary.

4. Slope of $\overline{QU}$ is $m_{\overline{QU}} = \frac{-1-(-1)}{3-5} = 0$; Slope of $\overline{DA}$ is $m_{\overline{DA}} = \frac{3-3}{6-(-2)} = 0$

Slope of $\overline{QD}$ is $m_{\overline{QD}} = \frac{-1-3}{-5-(-2)} = \frac{-4}{-3} = \frac{4}{3}$; Slope of $\overline{UA}$ is $m_{\overline{UA}} = \frac{-1-3}{3-6} = \frac{-4}{-3} = \frac{4}{3}$.

5. $QUAD$ is a parallelogram because opposite sides are parallel.

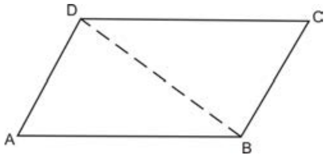
6. $QD = \sqrt{(-5-(-2))^2+(-1-3)^2} = \sqrt{(-3)^2+(-4)^2} = \sqrt{25} = 5$. We can conclude that $UA = 5$ also.

7. Since the opposite sides are parallel, $m\angle U = 180^\circ - m\angle Q = 180^\circ - 53^\circ = 127^\circ$.

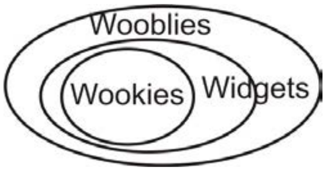
For the same reason, $m\angle A = 180^\circ - m\angle Q = 180^\circ - 53^\circ = 127^\circ$.

8.

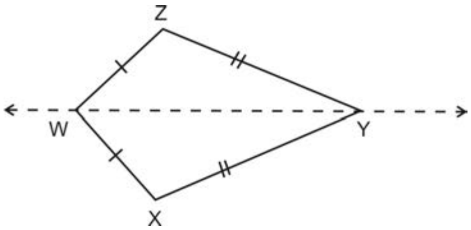
Table 6.3

| Statement | Reason |
|---|---|
| 1. $ABCD$ is a parallelogram. | 1. Given |
| 2. Draw diagonal $\overline{BD}$, such that:<br><br> | 2. Line Postulate |
| 3. $\overline{AB} \times \overline{DC}$ | 3. definition of parallelogram |
| 4. $\angle 1 \cong \angle 3$ | 4. Alternate Interior Angles Theorem |
| 5. $\overline{AD} \times \overline{BC}$ | 5. definition of parallelogram |
| 6. $\angle 2 \cong \angle 4$ | 6. Alternate Interior Angles Theorem |
| 7. $\overline{BD} \cong \overline{BD}$ | 7. Reflexive Property of Segment Congruence |
| 8. $\triangle ABD \cong \triangle CDB$ | 8. ASA Triangle Congruence Postulate |
| 9. $\angle A \cong \angle B$ | 9. CPCTC |
| 10. $m\angle 1 + m\angle 4 = m\angle ABC$<br>$m\angle 2 + m\angle 3 = m\angle ADC$ | 10.Angle Addition Postulate |
| 11. $m\angle ABC = m\angle ADC$ | 11. Substitution |
| 12. $\angle ABC \cong \angle ADC$ | 12. definition of congruent angles |

9.



10.



There is only one line of symmetry. It is through $W$ and $Y$. From the diagram, we can determine that $\angle X \cong \angle Z$.

## Using Parallelograms

1. $DG = 5$ cm (opposite sides are congruent), $DF = 7.25$ cm (opposite sides are congruent), $AD = 11$ cm (each diagonal bisects the other diagonal)

2. $a = 76^\circ$ because Corresponding Angles Postulate, $b = 180^\circ - 76^\circ = 104^\circ$ because consecutive interior angles of a parallelogram are supplementary.

3. Slope of $\overline{AD}$ is $m_{\overline{AD}} = \frac{4-1}{-1-(-4)} = \frac{3}{3} = 1$ ; Slope of $\overline{CB}$ is

$$m_{\overline{CB}} = \frac{-1-(-4)}{4-1} = \frac{3}{3} = 1$$

4. Slope of $\overline{DC}$ is $m_{\overline{DC}} = \frac{-4-1}{1-(-4)} = \frac{-5}{5} = -1$; Slope of $\overline{AB}$ is

$$m_{\overline{AB}} = \frac{-1-4}{4-(-1)} = \frac{-5}{5} = -1$$

5. $ABCD$ is a parallelogram because opposite sides are parallel. It is also a rectangle because consecutive sides are perpendicular.

6. The diagonals would intersect at $(0,0)$. Each vertex is a 90° rotation about the origin from the consecutive vertex.

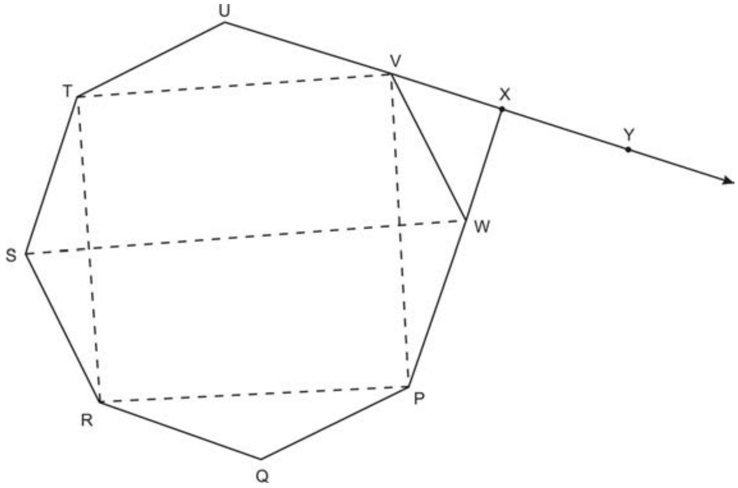
7. Since $PQRSTUVW$ is an octagon, the measure of each interior angle is 135°, so $m\angle RST = 135°$ (refer to item 8, section 2 for more information).

8. Each exterior angle in a regular octagon is 45° because $180° - 135° = 45°$. Therefore, $m\angle VWX = 45°$.

9. $m\angle WXY = 90°$. Each exterior angle is 45°, so $m\angle VXW = 90°$ by the Triangle Sum Theorem. Then, $m\angle WXY = 180° - m\angle VXW = 90°$.

10. $\triangle WVX$ is an isosceles right triangle.

11.



a. $PRTV$ is a parallelogram. More specifically, it is a square.

b. $STVW$ is a trapezoid.

c. $\triangle RST$ is an isosceles triangle.

## Proving Quadrilaterals are Parallelograms

1. a. $m\angle FBC = 180° - 160° = 20°$ (Linear Pair Postulate)

b. Since the sum of the interior angles of a triangle is $180^\circ, m\angle FBA = 180^\circ - (48^\circ + 86^\circ) = 46^\circ$.

c. Since $ABCD$ is a parallelogram, we know opposite angles are congruent. Therefore, $\angle ABC \cong \angle ADC$. Then, by the definition of congruent angles, $m\angle ABC = m\angle ADC$. By the Angle Addition Postulate, $m\angle ABC = m\angle FBC + m\angle FBA = 66^\circ$. Thus, by substitution, $m\angle ADC = 66^\circ$.

d. Since $ABCD$ is a parallelogram, we know consecutive angles are supplementary, so $m\angle ABC + m\angle BCD = 180^\circ$. Solving for $m\angle BCD$ yields $m\angle BCD = 180^\circ - m\angle ABC = 180^\circ - 66^\circ = 114^\circ$.

2. a. $AD = 9.5$ meters since opposite sides of a parallelogram are congruent.

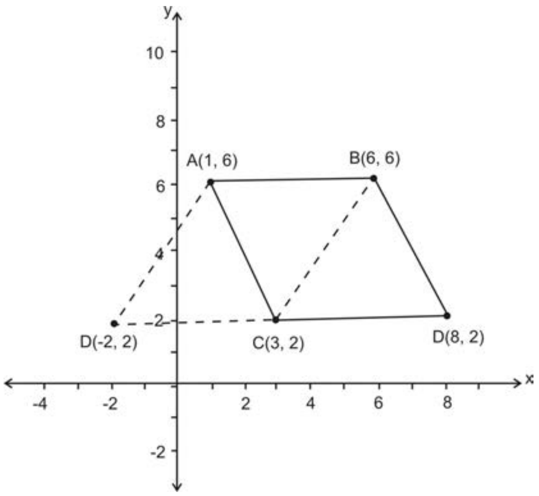
b. $DC = 4.5$ meters since opposite sides of a parallelogram are congruent.

3. a. $AF = \frac{1}{2}AC$ since the diagonals in a parallelogram bisect each other. So, $AF = \frac{1}{2}(8.1) = 4.05$ meters.

b. $BD = 2(BF) = 12$ meters since the diagonals in a parallelogram bisect each other.

4. Since $ABCD$ is a parallelogram, we know the opposite sides are both congruent and parallel. So, since $AB = |6 - 1| = 5$, then we know $CD = 5$ as well. We know the $y-$coordinate of $D$ is 2, so that means the side is horizontal. We can find the $x-$coordinate of $D$ by adding 5 to the $C$ $x-$coordinate or subtracting 5 from the $C$ $x-$coordinate. Thus, $D$ could be $(3 - 5, 2) = (-2, 2)$ or could be $(3 + 5, 2) = (8, 2)$.

5. If you look at the figure below, you will see the dotted version with $D(-2, 2)$ is named $ABCD$. You will also notice the solid-lined version with $D(8, 2)$ is named $ABDC$.



6. The side parallel to side $\overline{AC}$ is side $\overline{BD}$; they have the same slope. The slope of side $\overline{AC}$ can be found by $m_{\overline{AC}} = \frac{2-6}{3-1} = \frac{-4}{2} = -2, m_{\overline{BD}} = -2$.

7. $BD = \sqrt{(8-6)^2 + (2-6)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

8.

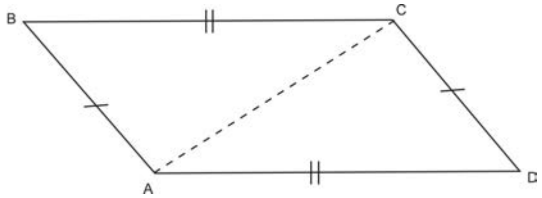


Table 6.4

| Statement | Reason |
|---|---|
| 1. $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ | 1. Given |
| 2. draw a diagonal $\overline{AC}$ | 2. Line Postulate |
| 3. $\overline{AC} \cong \overline{AC}$ | 3. Reflexive Property of Segment congruence |
| 4. $\triangle ABC \cong \triangle CDA$ | 4. SSS Triangle Congruence Postulate |
| 5. $\angle BAC \cong \angle DCA, \angle ACB \cong \angle CAD$ | 5. CPCTC |
| 6. $\overline{AB} \times \overline{DC}, \overline{AD} \times \overline{BC}$ | 6. Converse of Alternate Interior Angles Postulate |

9.

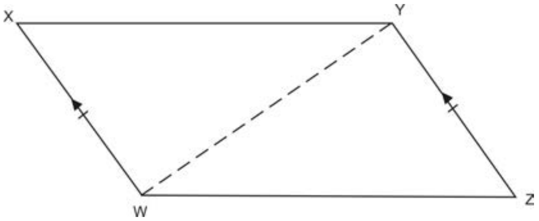
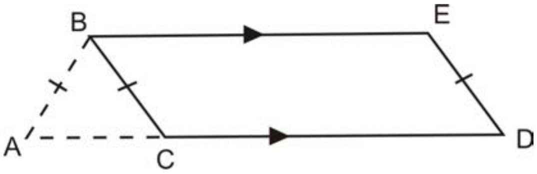


Table 6.5

| Statement | Reason |
|---|---|
| 1. $WXYZ$ with $\overline{WX} \cong \overline{ZY}, \overline{WX} \times \overline{ZY}$ | 1. Given |
| 2. draw a diagonal $\overline{WY}$ | 2. Line Postulate |
| 3. $\overline{WY} \cong \overline{WY}$ | 3. Reflexive Property of Segment congruence |
| 4. $\angle YWX \cong \angle WYZ$ | 4. Alternate Interior Angles Postulate |
| 5. $\triangle WXY \cong \triangle WZY$ | 5. SAS Triangle Congruence Postulate |
| 6. $\angle YWX \cong \angle WYX$ | 6.CPCTC |
| 7. $\overline{WZ} \times \overline{XY}$ | 7. Converse of Alternate Interior Angles Postulate |
| 8. $WXYZ$ is a parallelogram | 8. definition of a parallelogram |

10.



The only two quadrilaterals one could draw with these criteria are a parallelogram or an isosceles trapezoid.

## Rhombi, Rectangles, and Squares

1. a. $TC = 3.5$ cm since opposite sides are congruent.

b. $EC = 6.4$ cm since opposite sides are congruent.

2. a. $ET = 7.2$ cm since diagonals are congruent.

b. $RL = \frac{1}{2}RC = \frac{1}{2}(7.2) = 3.6$ cm because diagonals bisect each other.

3. a. $m\angle REC = 90°$ since $RECT$ is a rectangle.

b. $m\angle LTC = 61°$. First, $\angle REL$ and $\angle LEC$ are complementary, so $m\angle REL = 61°$. Then, since opposite sides are parallel, alternate interior angles are congruent, so $\angle REL \cong \angle LTC$.

4. a. $RB = 54$ inches since all sides in a rhombus are congruent.

b. $RS = \frac{1}{2}RM = \frac{1}{2}(52) = 26$ inches since diagonals bisect each other.

5. a. $m\angle RMO = 59°$ since opposite sides of a rhombus are parallel.

b. $m\angle RBM = 62°.m\angle RMB = 59°$, creating an isosceles triangle, $\triangle RBM$. Then, $m\angle RBM = 180 - 2(59) = 62°$.

6. Perimeter is $4(54) = 216$ inches since all the sides of a rhombus are congruent

7. $\overline{BO}$ is the perpendicular bisector of $\overline{RM}$.

8. Biconditional: A quadrilateral is a square if and only if it is a rhombus. This statement is false because a rhombus is not always a square. Refer to figure $ROMB$ in exercises 4 - 7 as an example.

9. A quadrilateral has 4 right angles if and only if it is a rectangle. This statement is true.

10. if-then: If a quadrilateral is a square, then it is a rectangle with 4 congruent sides.

biconditional: A quadrilateral is a square if and only if it is a rectangle with 4 congruent sides.

## Trapezoids

1. $m\angle ADC = 40°$ since the base angles of a trapezoid are congruent.

2. $m\angle BCD = 140°$ by the Same-Side Interior Theorem.

3. $m\angle RAP = 73° - 56° = 17°$ since the base angles of a trapezoid are congruent.

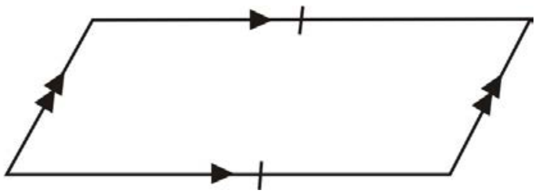
4. The diagonals of an isosceles trapezoid are congruent, so $AR = 11.5$ cm.

5. Opposite angles in an isosceles trapezoid are supplementary, so $m\angle ATR = 180° - 73° = 107°$.

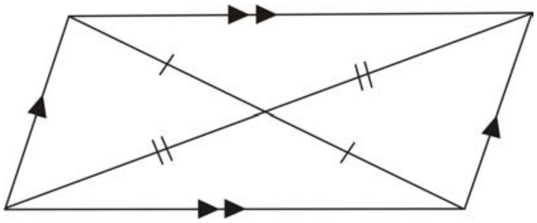
6. $m\angle MAE = 180° + 96° = 84°$. First, $A$ is the midpoint of $\overline{MY}$ and $E$ is the midpoint of $\overline{LT}$, so $\overline{AE}$ is the midsegment of the trapezoid. By the definition of a midsegment, $\overline{AE}$ is parallel to both bases. Then, by the Same-Side Interior Angles Theorem, $\angle LMY$ and $\angle MAE$ are supplementary.

7. $EA = \frac{15+21}{2} = 18$ cm

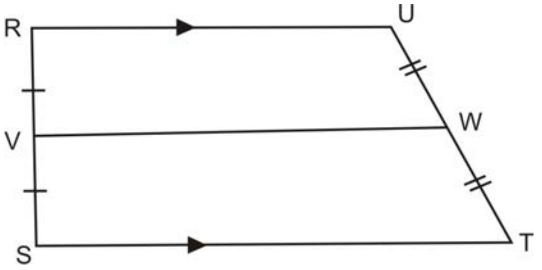
8. The parallel sides of a trapezoid cannot be congruent because then it would be a parallelogram.



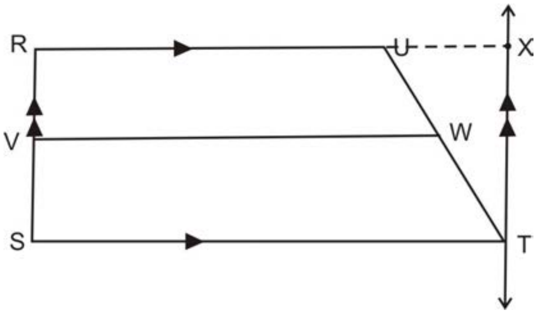
9. The diagonals of a trapezoid cannot bisect each other or it would be a parallelogram.



10. We will start with the following trapezoid $RSTU$ with midsegment $\overline{VW}$.



Then, we can use the Parallel Postulate to draw a line through $T$ that is parallel to $\overline{RS}$. The line intersects $\overline{RU}$ at $X$ and $\overline{VW}$ at $Y$ as shown.



By construction, $RSTX$ is a parallelogram. Since we have a parallelogram, we know $\overline{RX} \cong \overline{ST} \cong \overline{VY}$, so $RX = ST = VY$. We know by the Triangle Midsegment Theorem that $YW = \frac{1}{2}XU$ or $XU = 2YW$.

We need to show $VW = \frac{ST+RU}{2}$. Therefore, by the Segment Addition Postulate, $\frac{ST+RU}{2} = \frac{ST+RX+XU}{2}$. We can use some of the values above to see that we can edit this equation to $\frac{ST+RU}{2} = \frac{VY+VY+2YW}{2} = \frac{2VY+2YW}{2} = VY + YW$. Since $VY + YW = VW$, we have shown the midsegment is one-half the sum of the bases.

## Kites

1. $m\angle KIT = 160°$ since $\overline{IE}$ bisects $\angle KIT$.

2. $m\angle TEI = 28°$ since $\overline{IE}$ bisects $\angle TEK$.

3. We know the sum of $\triangle EKI = 180°$. We are given $m\angle KEI = 28°$. We also know $m\angle KIE = 80°$ since $\overline{IE}$ bisects $\angle KIT$. Then, $m\angle EKI = 180° - (28° + 80°) = 72°$.

4. The diagonals are perpendicular to each other, so $m\angle KCE = 90°$.

5. The diagonals bisect each other, so $KC = \frac{1}{2}KT = 4.1$ cm.

6. The vertex angles of kite $ABCD$ are $\angle BAD$ and $\angle BCD$.

7. $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

8. Diagonal $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$. Diagonal $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$.

9. $\angle CBD \cong \angle CDB, \angle ADC \cong \angle ABC, \angle BCA \cong DCA, \angle ABD \cong ADB, \angle BAC \cong \angle DAC$

10. The line of symmetry in the kite is along segment $\overline{AC}$.

11. If the diagonals were congruent, the figure would be a square. By definition, the length of the pairs of congruent sides must be different.

# Chapter 7

# TE Similarity - Solution Key

## 7.1 Similarity

### Ratios and Proportions

1. a. $\frac{\text{Milhone}}{\text{Suarez}} = \frac{32}{24} = \frac{4}{3}$

b. $\frac{\text{Cho}}{\text{Milhone}} = \frac{20}{32} = \frac{5}{8}$

c. $\text{Suarez} : \text{Milhone} : \text{Cho} = 24 : 32 : 20 = 6 : 8 : 5$

d. $\frac{\text{Suarez or Cho}}{\text{Total}} = \frac{44}{76} = \frac{22}{38} = \frac{11}{19}$

2. a. $16 : 16 \to 1 : 1$

b. $16 : 8 \to 2 : 1$

c. $8 : 16 \to 1 : 2$

d. $16 : 24 \to 2 : 3$

e. $\text{Area}_{\text{MNRQ}} = 16^2 = 256; \ \text{Area}_{\text{NPSR}} = 8 \times 16 = 128$

$256 : 128 = 2 : 1$

f. $\text{Area}_{\text{NPSR}} = 8 \times 16 = 128; \ \text{Area}_{\text{MNRQ}} = 16^2 = 256$

$128 : 256 = 1 : 2$

g. $\text{Area}_{\text{MNRQ}} = 16^2 = 256; \ \text{Area}_{\text{MPSQ}} = 24 \times 16 = 384$

$256 : 384 = 2 : 3$

3. If we let $x$ represent the unknown quantity, then we can use the Triangle Sum Theorem to find the measures of the angles.

$$\begin{aligned} 3x + 3x + 4x &= 180^\circ \\ 10x &= 180^\circ \\ x &= 18^\circ \end{aligned}$$

Then, the two smaller congruent angles have a measure of $3(18^\circ) = 54^\circ$ and the larger angle has a measure of $4(18^\circ) = 72^\circ$.

4. Since the ratio of the length to width is $3 : 5$, we can find those measures by

$$\begin{aligned} 3x(5x) &= 540 \\ 15x^2 &= 540 \\ x^2 &= 36 \\ x &= 6 \end{aligned}$$

So, the length is $3(6) = 18$ inches and the width is $5(6) = 30$ inches.

5. We are given that $a, b, c,$ and $d$ are real numbers where $b \neq 0, d \neq 0$. We are also given $ad = bc$. By the mupltiplication property of equality, we can multiply both sides of the equation by anything we wish, so we will chose $\frac{1}{bd}$.

$$ad \times \frac{1}{bd} = bc \times \frac{1}{bd} \quad \text{or} \quad \frac{ad}{bd} = \frac{bc}{bd}$$

If we separate the fractions, we get

$$\frac{a}{b} \times \frac{d}{d} = \frac{b}{b} \times \frac{c}{d}$$

As you can see, we have two fractions that are equivalent of 1 $\left(\frac{d}{d} \text{ and } \frac{b}{b}\right)$, so

$$\frac{a}{b} = \frac{c}{d}$$

6. We will use the cross-products property for each of these to determine which ones are equal.

a. $\frac{a}{b} = \frac{c}{d}$ simplifies to $ad = bc$, and $\frac{a}{d} = \frac{c}{b}$ simplifies to $ab = cd$. (NOT EQUAL)

b. $\frac{a}{b} = \frac{c}{d}$ simplifies to $ad = bc$, and $\frac{a}{c} = \frac{b}{d}$ simplifies to $ad = bc$. (EQUAL)

c. $\frac{a}{b} = \frac{c}{d}$ simplifies to $ad = bc$, and $\frac{b}{a} = \frac{d}{c}$ simplifies to $ad = bc$. (EQUAL)

d. $\frac{a}{b} = \frac{c}{d}$ simplifies to $ad = bc$, and $\frac{b}{c} = \frac{a}{d}$ simplifies to $ac = bd$. (NOT EQUAL)

7. We will use the cross-products property for each of these.

a. $\frac{6}{w} = \frac{4}{5} \Rightarrow 4w = 30 \Rightarrow w = 7.5$ b. $\frac{w}{3} = \frac{12}{w} \Rightarrow w^2 = 36 \Rightarrow w = \pm 6$ c. $\frac{3}{4} = \frac{w}{w+2} \Rightarrow 4w = 3(w + 2) \Rightarrow 4w = 3w + 6 \Rightarrow w = 6$

8. If we set up each ratio as $\frac{\text{miles}}{\text{gallons}}$, then our proportion would be $\frac{245}{8.2} = \frac{416}{x}$.

9. Solving for $x$ gives us $\frac{245}{8.2} = \frac{416}{x} \Rightarrow 245x = 3411.2 \Rightarrow 13.92$ gallons. (rounded to the nearest hundredth).

10. If we let $x$ represent the unknown quantity, then

$$\begin{aligned} 3x + 2x + 4x &= \$1,800,00 \\ 9x &= \$1,800,00 \\ x &= \$20,0000 \end{aligned}$$

Then, Rashid earns $4(\$20,000) = \$80,000$, Leon earns $2(\$20,000) = \$40,000$, and Maria earns $3(\$20,000) = \$60,000$.

## Properties of Proportions

1. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{10}{y} = \frac{x}{6}$ simplifies to $xy = 60$, so this proportion is false.

2. $\frac{10}{15} = \frac{6}{9}$ simplifies to $90 = 90$, so this proportion is true.

3. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{10}{y} = \frac{6}{x}$ simplifies to $10x = 6y$, so this proportion is false.

4. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{y}{6} = \frac{x}{10}$ simplifies to $6x = 10y$, so this proportion is true.

5. $\frac{9}{15} = \frac{6}{10}$ simplifies to $90 = 90$, so this proportion is true.

6. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{6}{x} = \frac{10}{y}$ simplifies to $6y = 10x$, so this proportion is false.

7. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{25}{15} = \frac{x}{y}$ simplifies to $25y = 15x$ or $5y = 3x$, so this proportion is false.

8. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{10}{16} = \frac{x}{x+y}$ simplifies to $10(x+y) = 16x \rightarrow 10x + 10y = 16x \rightarrow 4x = -10y$. This statement is false.

9. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{33}{9} = \frac{x+2y}{y}$ simplifies to $33y = 9(x+2y) \rightarrow 33y = 9x + 19y \rightarrow 14y = 9x$. This statement is false.

10. We are given $\frac{10}{6} = \frac{x}{y}$ which simplifies to $10y = 6x$. $\frac{4}{6} = \frac{y-x}{y}$ simplifies to $4y = 6(y-x) \rightarrow 4y = 6y - 6x \rightarrow -2y = -6x \rightarrow y = -3x$. This statement is false.

11. We are given $\frac{a}{b} = \frac{c}{d}$ , where $b \neq 0, d \neq 0$. If we cross multiply, we get $ad = bc$. We can show the distributive property with $a(c+d) = ac + ad$. If we substitute $bc$ in for $ad$, then our equation becomes $a(c+d) = ac + bc$. Now we can factor out $c$ on the right side, giving us $a(c+d) = c(a+b)$. If we apply the Cross Multiplication Theorem, we get $\frac{a}{a+b} = \frac{c}{c+d}$, which is what we were trying to show.

12. This proof will be very similar to the proof from item 11. We are given $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$. If we cross multiply, we get $ad = bc$. We can show the distributive property with $d(a-b) = ad - bd$. If we substitute $bc$ in for $ad$, then our equation becomes $d(a-b) = bc - bd$. Now we can factor out $b$ on the right side, giving us $d(a-b) = b(c-d)$. If we apply the Cross Multiplication Theorem, we get $\frac{a-b}{b} = \frac{c-d}{d}$, which is what we were trying to show.

## Similar Polygons

1. This statement is true because the three angles in any equilateral triangle will be congruent to any three angles in another equilateral triangle.

2. This statement is false. One could have an isosceles triangle with angle measures of $45^\circ - 45^\circ - 90^\circ$ or an isosceles triangle with angle measures of $20^\circ - 20^\circ - 140^\circ$ (among many examples).

3. This statement is false. One could have a rectangle with dimensions of $2 \times 3$, or a square (which is also a rectangle) with dimensions $2 \times 2$ (among other examples).

4. This statement is false. A rhombus has four congruent sides, but the angles could be 4 right angles or 2 $45^\circ$ angles and 2 $135^\circ$ angles.

5. This statement is true because the four angles in any square will be congruent to any four angles in another square.

6. This statement is true because by definition, congruent polygons have corresponding angles congruent, which gives an angle ratio of $1:1$.

7. This statement is false. Similar polygons have corresponding proportional sides, but not necessarily congruent sides.

8. Since $ABCD$ is a rectangle, then $\overline{AB} \cong \overline{CD}$, so $AB = CD = 45$.

9. We are given $ABCD \sim$ rectangle $AMNP$. The ratio of the sides can be found by $\frac{AM}{AB} = \frac{30}{45} = \frac{2}{3}$, so we can find $BC$ by using this same ratio. Since $AMNP$ is a rectangle, then $\overline{AP} \cong \overline{MN}$, so $AP = MN = 40$. Then $\frac{40}{BC} = \frac{2}{3}$. Cross multiplying is $2BC = 120$ or $BC = 60$.

10. From item 8, we know $AB = 45$. From the Segment Addition Postulate, $AM + MB = AB$. Substituting gives us $30 + MB = 45$ which yields $MB = 15$.

11. Since $ABCD$ is a rectangle, $AD = 60$ using the argument from item 8 and the result from item 9. From the Segment Addition Postulate, $AP + PD = AD$. Substituting gives us $40 + PD = 60$ which yields $PD = 20$.

12. Since the triangles are congruent, the scale factor is $1:1$.

13. $P_{\triangle PQR} = 2 + 3 + 3 = 8$ units

14. $P_{\triangle TSR} = 6 + 6 + 4 = 16$ units

15. The ratio of the perimeters of the triangles is $\frac{P_{\triangle PQR}}{P_{\triangle TSR}} = \frac{1}{2}$

16. $\frac{PQ}{TS} = \frac{3}{6} = \frac{1}{2}$; $\frac{RQ}{RS} = \frac{3}{6} = \frac{1}{2}$; $\frac{PR}{TR} = \frac{2}{4} = \frac{1}{2}$, so all sides are proportional. We also know $\angle PRQ \cong \angle TRS$ by the Vertical Angles Theorem.

$\angle QPR \cong \angle RTS$; $\angle PQR \cong \angle RST$ by Alternate Interior Angles Theorem. Since all corresponding angles are congruent and all corresponding sides are proportional, $\triangle PQR \times \triangle TSR$.

17. a. $\overline{MN} \times \overline{AC}$ b. $\angle BMN \cong \angle BAC, \angle BNM \cong \angle BCA$ c. $\triangle BNM \times \triangle BCA$ d. The ratio of the sides of the smaller triangle to the larger triangle is $\frac{1}{2}$, so that is the ratio of the perimeters. The ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle is $\frac{1}{2}p$.

e. The ratio of the sides of $\triangle BNM$ and $\triangle BCA$ is $\frac{1}{2}$, therefore the ratio of the area of the smaller to the area of the larger one is $\frac{1}{4}$, so the area of $\triangle BNM$ is $\frac{1}{4}(100) = 25$ square units. Then, by area addition, $\text{Area}_{\triangle \text{ABC}} = \text{Area}_{\triangle \text{BNM}} + \text{Area}_{\text{AMNC}}$ , so $\text{Area}_{\text{AMNC}} = 100 - 25 = 75$ square units.

## Similarity by AA

1. $\triangle ABE \cong \triangle CDE$

2. $\angle ABE \cong \angle CDE; \angle BAE \cong \angle DCE$ by Alternate Interior Angles Theorem. We also know $\angle AEB \cong$

$\angle CED$ by the Vertical Angles Theorem. So, $\triangle ABE \cong \triangle CDE$ by $AAA$ Similarity Postulate or $AA$ Similarity Theorem.

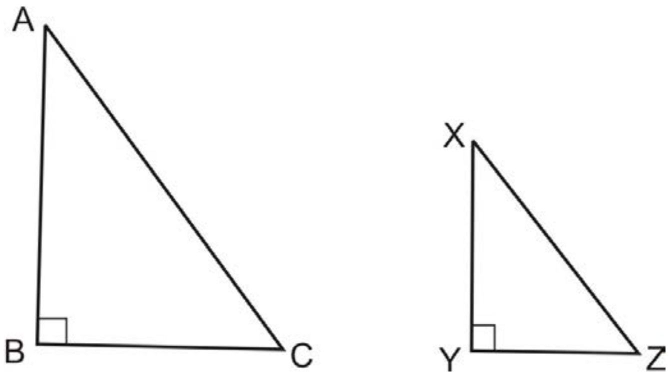
3. $\frac{AB}{CD} = \frac{BE}{DE} = \frac{AE}{CE}$

4. $\triangle DEA$ may not be similar to $\triangle BCE$.

5. Using the proportions from item 3, $\frac{AB}{CD} = \frac{AE}{CE}$ which gives us $\frac{10}{22} = \frac{7}{CE}$. Therefore, by using cross multiplication, $CE = 15.4$. Them by the Segment Addition Postulate, $AE + CE = AC$. Substitution gives us $7 + 15.4 = 22.4$

6. By $AA$, $\triangle ABC \times \triangle TRS$ which tells us the sides are proportional. More specifically, $\frac{AB}{TR} = \frac{BC}{RS} = \frac{AC}{TS}$. Using this proportion, we have $\frac{8}{6} = \frac{k}{RS}$ which yields $RS = \frac{6k}{8} = \frac{3}{4}k$.

7.Consider the right triangles $\triangle ABC$ and $\triangle XYZ$ where $\angle B$ and $\angle Y$ are right angles and $\angle A$ and $\angle X$ are congruent acute angles as seen in the figure below.



Since we have two angles in one triangle congruent to two corresponding angles in another triangle, by $AA$, the triangles are similar.

8. $\triangle ABC \times \triangle EDC$

9. We know the triangles are similar because they are both right triangles, so $\angle B \cong \angle D$. We also know $\angle BCA \cong \angle DCE$ by the Vertical Angles Theorem. This makes the triangles similar by $AA$.

10. Since the triangles are similar, the sides are proportional. Thus, $\frac{AB}{ED} = \frac{CB}{CD}$. If we substitute the known values, we get $\frac{30}{80} = \frac{50}{CD}$.

11. $\frac{30}{80} = \frac{50}{CD} \Rightarrow 30CD = 4000 \Rightarrow CD \approx 133$. The river is approximately 133 feet wide.

12. The model is a fair one for the sides and width of a river. One must account for varying widths and different places in the river, and the curvature of rivers in general, but for the most part, sides of rivers are straight enough to fit the model.

## Similarity by SSS and SAS

1. The triangles are not congruent because one is much larger than the other.

2. The triangles are congruent because each of the pairs of corresponding sides are proportional (SSS).

3. We much first convert each of the sides of the larger triangle to inches, so 3 feet = 36 inches and 4 feet = 48 inches. Therefore, the scale factor is $\frac{1}{12}$.

4. It is not necessary to compare the included side of two triangles because of $AA$ Similarity.

5. The sides are not proportional, so $\triangle ABC$ is not similar to $\triangle MNP$.

6. $\triangle MNP$ is not a valid triangle, so the triangles are not similar.

7. The sides are not proportional, so $\triangle ABC$ is not similar to $\triangle MNP$.

8. There is not enough information to solve this item.

9. For triangles to be similar, all corresponding angles must be congruent; therefore, $\triangle ABC$ is not similar to $\triangle MNP$.

10. Since all corresponding sides are proportional, $\triangle ABC \cong \triangle MNP$ by SSS.

11. a. Yes

b. Yes

c. Since all three pairs of corresponding angles are congruent, the triangles are similar by $AAA$.

## Proportionality Relationships

1. $\triangle ABD \cong \triangle ACE$

2. $\frac{AB}{BC} = \frac{AD}{DE}$

3. $\frac{AB}{AD} = \frac{BC}{DE}$

4. $\frac{AB}{AC} = \frac{AD}{AE}$

5. $\frac{AC}{AE} = \frac{BC}{DE}$

6. Since $k, m$, and $n$ are parallel, then $\frac{15}{x} = \frac{10}{15} \Rightarrow 10x = 225 \Rightarrow x = 22.5$.

7. Since $k, m$, and $n$ are parallel, then $\frac{6}{16} = \frac{x}{30} \Rightarrow 16x = 180 \Rightarrow x = 11.25$.

8. $\frac{10}{16} = \frac{y}{30} \Rightarrow 16x = 300 \Rightarrow x = 18.75$

9. First, you would position the endpoint of the slanted line on the first horizontal line. Then, you would place the endpoint on the other end of the slanted line on the seventh horizontal line. You would then connect your points on the slanted segment, thus dividing the original segment into seven congruent parts.

## Similarity Transformations

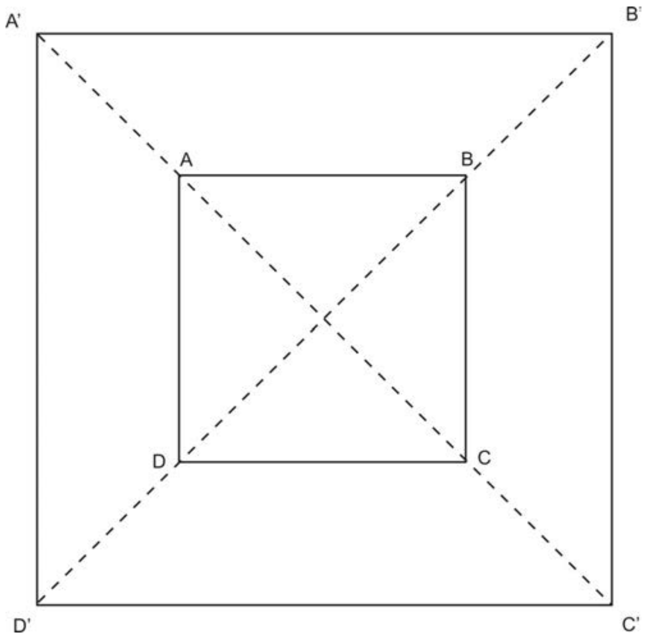
Table 7.1

| | Center | Scale Factor | Given Point | Image of Given Point |
|---|---|---|---|---|
| 1. | $C$ | 2 | $B$ | $A$ |
| 2. | $A$ | 0.5 | $C$ | $B$ |
| 3. | $C$ | 3 | $D$ | $F$ |
| 4. | $E$ | 2 | $D$ | $C$ |
| 5. | $F$ | $\frac{1}{3}$ | $C$ | $E$ |
| 6. | $B$ | 1 | $A$ | $A$ |
| 7. | $C$ | $\frac{2}{3}$ | $F$ | $E$ |
| 8. | $C$ | 0.5 or $\frac{1}{2}$ | $E$ | $D$ |

Table 7.1: (continued)

| | Center | Scale Factor | Given Point | Image of Given Point |
|---|---|---|---|---|
| 9. | $C$ | 0.75 or $\frac{3}{4}$ | $A$ | Midpoint of $\overline{AB}$ |
| 10. | $F$ | $\frac{5}{6}$ | $C$ | Midpoint of $\overline{CD}$ |

11.



12. The scale factor is 1 since all the corresponding pieces of the figures are congruent (definition of congruent figures).

13. All images of points will be at the center of the dilation.

14. We will begin by assigning the coordinates of $O$ to be $(0, 0)$. Then,

$$m_{\overline{OA}} = \frac{n-0}{m-0} = \frac{n}{m}; m_{\overline{OB}} = \frac{kn-0}{km-0} = \frac{kn}{km} = \frac{n}{m}$$
$$m_{\overline{AB}} = \frac{kn-n}{km-m} = \frac{n(k-1)}{m(k-1)} = \frac{n}{m}, k \neq 1$$

The segments have the same endpoints and the same slope, so the points are collinear.

15. The coordinates of $A''(45, -18)$.

## Self-Similarity

1. The number of segments in the second level is 4, and the third level is 8.

2. Each segment would be $\frac{1}{9}$ units long.

3. The fourth level will have $2^4$ segments $= 16$.

4. Level ten has $2^{10} = 1024$.

5. The generic formula is $2^n$ where $n$ is the level number.

6. Since the starting length is $S$, you divide the length by $3^n$, so $\frac{S}{3^n}$.

Refer to webpage:

http://north.ecc.edu/marinom/MedicalSTEP/Fractals%20Activity.pdf

7. There are 9 un-shaded triangles in level 2.

8. There are 27 un-shaded triangles in level 3.

9. In the generic level, $n$, there are $3^n$ un-shaded triangles.

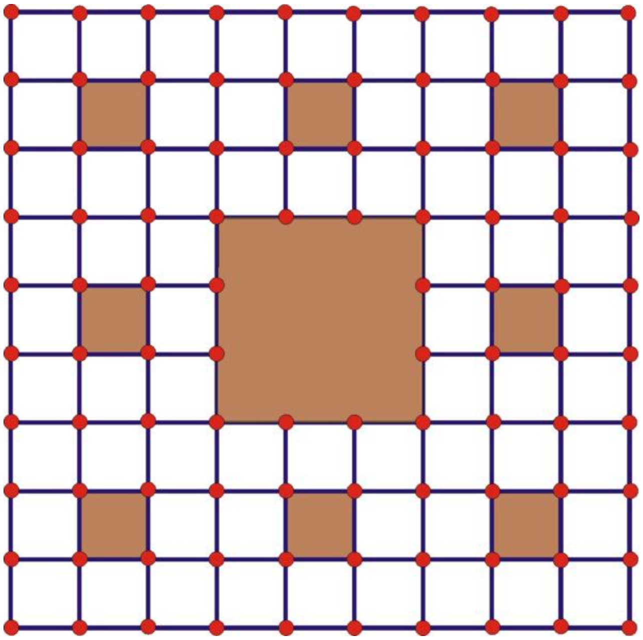
10. Total area of un-shaded part is $\frac{3}{4}$.

11. In level 2, the area of the un-shaded portion is $\frac{9}{16}$.

12. In level $n$, the formula is $\left(\frac{3}{4}\right)^n$, where $n$ is the level number.

13. The midsegments of a triangle divide it into four congruent triangles. Each of these new triangles is similar to the original triangle.

14.



# Chapter 8

# TE Right Triangle Trigonometry - Solution Key

## 8.1 Right Triangle Trigonometry

### The Pythagorean Theorem

1. $d = \sqrt{(-2-(-5))^2 + (-1-(-5))^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$

2. To check if it makes a Pythagorean triple, use the numbers in the Pythagorean Theorem where the largest number represents the hypotenuse.

$$20^2 = 12^2 + 16^2 \Rightarrow 400 = 144 + 256 \Rightarrow 400 = 400$$

Since both sides of the equation are the same, these numbers satisfy the Pythagorean Thm, so they are the sides of a right triangle.

3. $p^2 = 8^2 + 15^2 \Rightarrow p = \sqrt{64+225} \Rightarrow p = \sqrt{289} \Rightarrow p = 17$ cm

4. To check if it makes a Pythagorean triple, use the numbers in the Pythagorean Theorem where the largest number represents the hypotenuse.

$$35^2 = 26^2 + 13^2 \Rightarrow 1225 = 676 + 169 \Rightarrow 1225 = 845$$

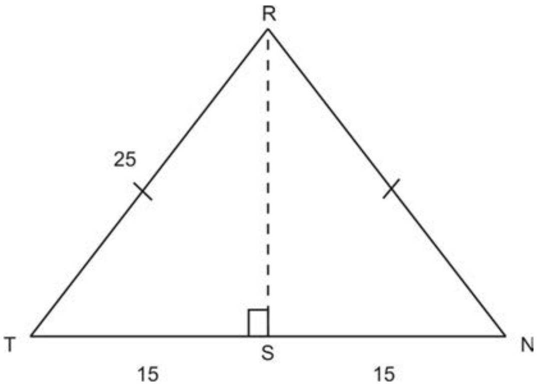
Since the two sides of the equation are not the same, these sides cannot make a right triangle or satisfy the Pythagorean Theorem.

5. $d = \sqrt{(9-1)^2 + (4-9)^2} = \sqrt{8^2 + (-5)^2} = \sqrt{64+25} = \sqrt{89}$

6. $m^2 = 9^2 + 12^2 \Rightarrow m = \sqrt{81+144} \Rightarrow p = \sqrt{225} \Rightarrow p = 15$ in

7. $d = \sqrt{(6-(-3))^2 + (7-5)^2} = \sqrt{9^2 + 2^2} = \sqrt{81+4} = \sqrt{85}$

8. We need to draw the height of the triangle from $R$ to a point on $\overline{NT}$ that we will call $S$. Since the triangle is isosceles, the height bisects the base.



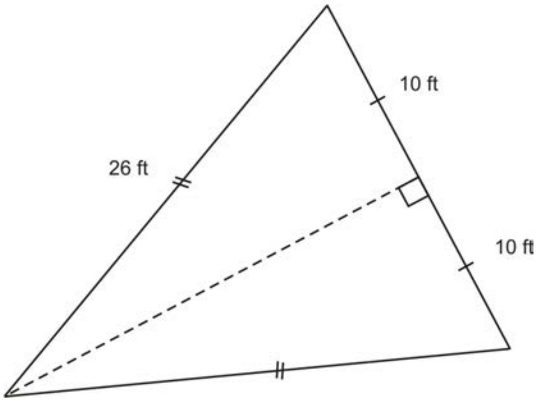
Then, we have a right triangle with a hypotenuse of 25 mm and one leg of 15 mm. We can calculate the height ($RS$), which is the missing side of the right triangle using the Pythagorean Theorem.

$$25^2 = RS^2 + 15^2 \Rightarrow RS = \sqrt{25^2 - 15^2} \Rightarrow RS = \sqrt{400} = 20 \text{ mm}$$

The height of $\triangle TRN$ is 20 mm and the base is 30 mm, so the area is

$$A = \frac{1}{2}bh = \frac{1}{2}(20)(30) = 300 \text{ mm}^2$$

9. The height of the triangle from the vertex angle to a point on the base. Since the triangle is isosceles, the height bisects the base.



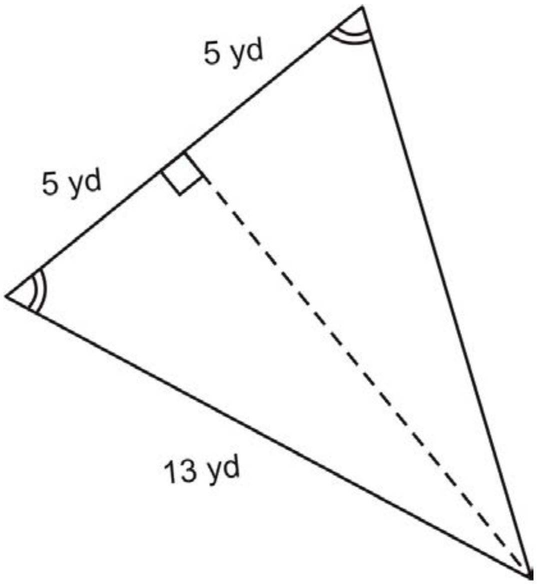
Then, we have a right triangle with a hypotenuse of 26 ft and one leg of 10 ft. We can calculate the height, which is the missing side of the right triangle using the Pythagorean Theorem.

$$26^2 = (\text{height})^2 + 10^2 \Rightarrow \text{height} = \sqrt{26^2 - 10^2} \Rightarrow \text{height} = \sqrt{576} = 24 \text{ ft}$$

The height of the triangle is 24 ft and the base is 20 ft, so the area is

$$A = \frac{1}{2}bh = \frac{1}{2}(24)(20) = 240 \text{ ft}^2$$

10. The height of the triangle from the vertex angle to a point on the base. Since the triangle is isosceles, the height bisects the base.



Then, we have a right triangle with a hypotenuse of 13 yd and one leg of 5 yd. We can calculate the height, which is the missing side of the right triangle using the Pythagorean Theorem.

$$13^2 = (\text{height})^2 + 5^2 \Rightarrow \text{height} = \sqrt{13^2 - 5^2} \Rightarrow \text{height} = \sqrt{144} = 12 \text{ yd}$$

The height of the triangle is 12 yd and the base is 13 yd, so the area is

$$A = \frac{1}{2}bh = \frac{1}{2}(13)(12) = 78 \text{ yd}^2$$

11. We can find the area of the inner square by squaring the length of the side $c, A = c^2$. We could also calculate the area of the larger square and subtract out the inner square. We should get the same result.

$$A_{\text{larger}} = (a+b)^2 = a^2 + 2ab + b^2$$

The larger square is made up of 4 small right triangles, each with an area of $\frac{1}{2}ab$ or $\frac{ab}{2}$.

With 4 of them, $4\left(\frac{ab}{2}\right) = 2ab$.

We can now subtract the area of the yellow squares from the area of the larger square to get the area of the smaller square.

$$A_{\text{smaller}} = a^2 + 2ab + b^2 - 2ab = a^2 + b^2$$

We have now shown that $c^2 = a^2 + b^2$.

## Converse of the Pythagorean Theorem

1. $15^2 = 9^2 + 12^2 \rightarrow 225 = 81 + 144 \rightarrow 225 = 225$. This shows that $9, 12$, and 15 are sides of a right triangle.

2. $8^2 = 7^2 + 7^2 \rightarrow 64 = 49 + 49 \rightarrow 64 < 98$. Since the sum of the squares of the legs is greater than the square of the longest side, this is an acute triangle.

3. $10^2 = 4^2 + 8^2 \rightarrow 100 = 16 + 64 \rightarrow 100 > 80$. Since the square of the longest side is greater than the sum of the squares of the shorter sides, this triangle is an obtuse triangle.

4. $23^2 = 22^2 + 10^2 \rightarrow 529 = 484 + 100 \rightarrow 529 < 584$. Since the sum of the squares of the legs is greater than the square of the longest side, this is an acute triangle.

5. $35^2 = 28^2 + 21^2 \rightarrow 1225 = 784 + 441 \rightarrow 1225 = 1225$. Since the sum of the squares of the legs is equal the square of the longest side, this is an acute triangle.

6. $14^2 = 12^2 + 10^2 \rightarrow 196 = 144 + 100 \rightarrow 196 < 244$. Since the sum of the squares of the legs is greater than the square of the longest side, this is an acute triangle.

7. $30^2 = 18^2 + 15^2 \rightarrow 900 = 324 + 225 \rightarrow 900 > 549$. Since the square of the longest side is greater than the sum of the squares of the shorter sides, this triangle is an obtuse triangle.

8. $110^2 = (\sqrt{75})^2 + 5^2 \rightarrow 12,100 = 75 + 25 \rightarrow 12,100 > 100$. Since the square of the longest side is greater than the sum of the squares of the shorter sides, this triangle is an obtuse triangle.

9. The longest side is $\sqrt{13} \approx 3.61$, so it is the longest side, thus should be used as the hypotenuse ($c$). Then the other sides would be the legs.

10. a. Since two of the sides of the triangle are congruent, it is an isosceles triangle. With that, we know the base angles are congruent, so $2(m\angle A) = 90^\circ \Rightarrow m\angle A = 45^\circ$

b. $(7\sqrt{2})^2 = 7^2 + 7^2 \rightarrow 98 = 49 + 49 \rightarrow 98 = 98$. Since the sum of the squares of the legs is equal the square of the longest side, this is an acute triangle. Since the longest side is opposite $\angle B, m\angle B = 90^\circ$.

## Using Similar Right Triangles

1. $\triangle DEF \times \triangle EGF \times \triangle DGE$

2. geometric mean $= \sqrt{1 \cdot 4} = \sqrt{4} = 2$ inches

3. geometric mean $= \sqrt{3 \cdot 3} = \sqrt{9} = 3$ centimeters

4. $\triangle MNO \times \triangle PNM \times \triangle PMO$

5. $h = \sqrt{4 \cdot 9} = \sqrt{36} = 6$

6. $d = \sqrt{3 \cdot 8} = \sqrt{24} = 2\sqrt{6} \approx 4.9$ millimeters

7. geometric mean $= \sqrt{4 \cdot 8} = \sqrt{32} = 4\sqrt{2} \approx 5.7$ yards

8. $a = \sqrt{5 \cdot 10} = \sqrt{50} = 5\sqrt{2} \approx 7.1$ feet

9. $g = \sqrt{7 \cdot 13} = \sqrt{91} = 9.5$ inches

10. $g = \sqrt{6 \cdot 13} = \sqrt{78} = 8.8$ inches

11. $k = \sqrt{6 \cdot 7} = \sqrt{42} = 6.5$ inches. You could also find $k$ using the Pythagorean Theorem.

12. $a = \sqrt{10 \cdot 8} = \sqrt{80} = 4\sqrt{5}$ centimeters

## Special Right Triangles

1. The angles of the two congruent triangles would be $30° - 60° - 90°$.

2. The angles of the two congruent triangles would be $45° - 45° - 90°$.

3. The length of the hypotenuse is 2(short leg) $= 2(5) = 10$ inches.

4. The length of the hypotenuse is $\sqrt{2}$ (leg) $= \sqrt{2}\ (11) = 11\sqrt{2} \approx 15.6$ centimeters.

5. It is probably easier to find the length of the short leg first. The leg of the short leg is $1 : 2$, so $\frac{1}{2} = \frac{x}{12} \to 2x = 12 \to x = 6$ miles. Then, the longer leg is $\frac{\sqrt{3}}{1} = \frac{x}{6} \to x = 6\sqrt{3} \approx 10.4$ miles.

6. The ratio of the legs of an isosceles triangle to the hypotenuse of the triangle is $1 : \sqrt{2}$, so the length of the legs is 3 millimeters.

7. The relationship of the long leg to the short leg is $\frac{\text{l.leg}}{\text{s.leg}} = \frac{\sqrt{3}}{1}$, so we can substitute our values in to get $\frac{14\sqrt{3}}{\text{s.leg}} = \frac{\sqrt{3}}{1}$. Crosss multiplying and simplifying gives short leg $= 14$ feet.

8. The ratio of the legs of an isosceles triangle to the hypotenuse of the triangle is $1 : \sqrt{2}$, so the length of the legs is 5 feet.

9. Cutting a square across the diagonal creates two isosceles triangles, so the ratio of legs to hyp is $1 : \sqrt{2}$. If the side is 3 feet, then the hyp is $3\sqrt{2} \approx 4.2$ feet.

10. The base of the triangle is 6 inches. If we draw a height to the base, we can find it because we have created a $30° - 60° - 90°$ right triangle with the short leg $= 3$ inches (half of one side). The long leg (which is the height of our triangle) is $3\sqrt{3} \approx 5.2$ inches.

The area of the triangle is $A = \frac{1}{2}(6)(3\sqrt{3}) \approx 15.6$ square inches.

## Tangent Ratio

1. The side opposite $\angle G$ is 8 mm.

2. Not including the hypotenuse, the side adjacent to $\angle G$ is 6 mm.

3. The hypotenuse is 10 mm.

4. $\tan \angle G = \frac{8}{6} = 1.33$

5. $\tan \angle H = \frac{6}{8} = 0.75$ 6. $m\angle C = 90 - 58 = 32°$ 7. $m\angle H = \tan^{-1}\left(\frac{8}{8}\right) = 45°$ 8. $\tan \angle R = \frac{24}{7} = 3.43$ **[solution in text is reciprocal]**

9. $\tan \angle S = \frac{7}{24} = 0.29$ 10. $m\angle E = 90°$ because it is a right angle.

## Sine and Cosine Ratios

1. $\sin \angle V = \frac{12}{13} \approx 0.92$

2. $\cos \angle V = \frac{5}{13} = 0.38$

3. $\cos \angle U = \frac{12}{13} = 0.92$

4. $\sin \angle O = \frac{12}{15} \approx 0.80$

5. $\cos \angle O = \frac{9}{15} \approx 0.60$

6. $\sin\angle M = \frac{9}{15} \approx 0.60$

7. $m\angle H = \cos^{-1}\left(\frac{15}{21.21}\right) = 45°$

8. $\sin\angle S = \frac{15}{17} = \approx 0.88$

9. $\cos\angle S = \frac{8}{17} \approx 0.47$

10. $m\angle E = \sin^{-1}\left(\frac{8.66}{10}\right) = 60°$

## Inverse Trigonometric Ratios

1. $m\angle G = \text{arccos}0.53 = 58°$
2. $m\angle V = \text{arctan}2.25 \approx 66.04°$
3. $m\angle B = \arccos\left(\frac{4}{8}\right) = 60°$
4. $m\angle M = \arcsin(0.978) = 77.96°$
5. $m\angle F = \arcsin\left(\frac{47}{50}\right) = 70.05°$
6. $m\angle L = \text{arctan}1.04 \approx 46.23°$
7. $m\angle D = \arccos(0.07) = 85.99°$
8. $m\angle M = \arctan\left(\frac{60}{2}\right) = 88.09°$
9. $m\angle D = \arccos\left(\frac{7}{18}\right) = 67.11°$
10. $m\angle Z = \arcsin\left(\frac{1}{10}\right) = 5.74°$

## Acute and Obtuse Triangles

1. $BC = \sqrt{7.5^2 + 10^2 - 2(7.5)(10)\text{cos}(53°)} = 8.12$ m
2. $\frac{8.12}{\sin 53°} = \frac{7.5}{\sin C} \Rightarrow 8.12\sin C = 5.99 \Rightarrow C = \arcsin\left(\frac{5.99}{8.12}\right) = 47.53°$
3. $\frac{6}{\sin 66°} = \frac{4.88}{\sin F} \Rightarrow 6\sin C = 4.46 \Rightarrow C = \arcsin\left(\frac{4.46}{6}\right) = 48.02°$
4.

$$\begin{aligned}6^2 &= 5^2 + 4^2 - 2(5)(4)\cos I\\ 36 &= 41 - 41\cos I\\ I &= \arccos\left(\frac{-5}{-40}\right) = 82.82°\end{aligned}$$

5. $KL = \sqrt{13^2 + 14^2 - 2(13)(14)\cos(75°)} = 16.46$ inches
6. $\frac{1.8}{\sin 20°} = \frac{2}{\sin O} \Rightarrow 1.8\sin O = 0.6840 \Rightarrow O = \arcsin\left(\frac{0.6840}{1.8}\right) = 22.3°$
7.

$$\begin{aligned}9^2 &= 10.5^2 + 14^2 - 2(10.5)(14)\cos P\\ 81 &= 306.25 - 294\cos P\\ P &= \arccos\left(\frac{-225.25}{-294}\right) = 140°\end{aligned}$$

8.

$$\begin{aligned} 10.5^5 &= 9^2 + 14^2 - 2(9)(14)\cos Q \\ 110.25 &= 277 - 252\cos Q \\ Q &= \arccos\left(\frac{-166.75}{-252}\right) = 48.57^\circ \end{aligned}$$

9.

$$\begin{aligned} TU &= \sqrt{7.5^2 + 5^2 - 2(7.5)(5)\cos 85^\circ} = 8.64 \text{ ft} \\ \frac{8.64}{\sin 85^\circ} &= \frac{5}{\sin T} \Rightarrow 8.64\sin T = 4.98 \Rightarrow T = \arcsin\left(\frac{4.98}{8.64}\right) = 35.20^\circ \end{aligned}$$

10.

$$\begin{aligned} 18^2 &= 15^2 + 13.5^2 - 2(15)(13.5)\cos W \\ 324 &= 407.25 - 405\cos W \\ W &= \arccos\left(\frac{-83.25}{-405}\right) = 78.14^\circ \end{aligned}$$

10. Since we are given 3 sides of the triangle, we will use the Law of Cosines.

$$\begin{aligned} XY^2 &= WX^2 + WY^2 - 2(WX)(WY)(XY)\cos W \Rightarrow \\ 18^2 &= 15^2 + 13.5^2 - 2(15)(13.5)(18)\cos W \Rightarrow \\ 324 &= 225 + 182.25 - 405\cos W \Rightarrow \\ -83.25 &= -405\cos W \\ 0.2056 &= \cos W \\ W &= \arccos(0.2056) = 78.14^\circ \end{aligned}$$